Applied Mathematical Sciences

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With 42 Illustrations

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To Silvana

PREFACE

Statistical mechanics may be naturally divided into two branches, one dealing with equilibrium systems, the other with nonequilibrium systems. The equilibrium properties of macroscopic systems are defined in principle by suitable averages in well-defined Gibbs's ensembles. This provides a framework for both qualitative understanding and quantitative approximations to equilibrium behaviour. Nonequilibrium phenomena are much less understood at the present time. A notable exception is offered by the case of dilute gases. Here a basic equation was established by Ludwig Boltzmann in 1872.

The Boltzmann equation still forms the basis for the kinetic theory of gases and has proved fruitful not only for a study of the classical gases Boltzmann had in mind but also, properly generalized, for studying electron transport in solids and plasmas, neutron transport in nuclear reactors, phonon transport in superfluids, and radiative transfer in planetary and stellar atmospheres. Research in both the new fields and the old one has undergone a considerable advance in the last thirty years.

In the last ten years, a new wave of interest has surrounded the Boltzmann equation, stemming from its unique role in the theory of time-dependent phenomena in large systems. In fact, the Boltzmann equation appears as a prototype of a reduced description taking into account only partial information about the underlying microscopic state (fully described by the coordinate and momenta of all the molecules), but nevertheless undergoing an autonomous time evolution. Thus the problem of the rigorous derivation of the Boltzmann equation from the microscopic description has attracted a certain amount of interest among physicists and mathematicians. This, in turn, has revived the interest in the theory of existence and uniqueness of the solutions of the Boltzmann equation, since this problem has proved to be intimately tied with the previous one.

This justifies the appearance of the present book (which tries to present a unified approach to the problems arising in the different fields mentioned above) by exploiting the similarities whenever they exist and underlining the differences when necessary. The main line of exposition, however, is tied to the classical equation established by Boltzmann, and hence the detailed descriptions of some applications almost exclusively refer to monatomic neutral gases. Appropriate references are given, however, to papers dealing with similar problems arising in other fields, with particular concern for neutron transport, gas mixtures, and polyatomic gases.

The material dealing with the basic properties and applications known before 1975 was covered in a previous book by the author*; thus, it was natural to incorporate in the present book the material of the first seven chapters of that book. This material is updated in an extensive Appendix, covering the developments from 1975 to 1987.

But the main feature of the book is Chapter VIII, completely rewritten and covering the important studies resulting from new mathematical approaches to the old problem of existence and uniqueness, and the new ones tied up with the question of validity. There is still no satisfactory proof for existence of solutions under general reasonable initial conditions. The material presented here indicates, however, that a great deal of progress has been achieved in recent times. There is no doubt that the better understanding resulting from this progress will be essential to the further penetration to be expected in the next few years.

It is hoped that the book will be useful as a textbook for an advanced course in kinetic theory and as a reference for mathematicians, physicists, and engineers interested in the kinetic theory of gases and its applications.

Milano, Italy
September 1987

CARLO CERCIGNANI

**Theory and Application of the Boltzmann Equation*, Scottish Academic Press, Edinburgh, 1975.

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