Undergraduate Texts in Mathematics

Undergraduate Texts in Mathematics

Apostol: Introduction to Analytic Number Theory. Second edition.
Armstrong: Groups and Symmetry.
Armstrong: Basic Topology.
Bak/Newman: Complex Analysis.
Banchoff/Wermer: Linear Algebra Through Geometry. Second edition.
Brémaud: An Introduction to Probabilistic Modeling.
Bressoud: Factorization and Primality Testing.
Brickman: Mathematical Introduction to Linear Programming and Game Theory.
Cederberg: A Course in Modern Geometries.
Childs: A Concrete Introduction to Higher Algebra.
Chung: Elementary Probability Theory with Stochastic Processes. Third edition.
Curtis: Linear Algebra: An Introductory Approach. Fourth edition.
Dixmier: General Topology.
Driver: Why Math?
Ebbinghaus/Flum/Thomas: Mathematical Logic.
Edgar: Measure, Topology, and Fractal Geometry.
Fischer: Intermediate Real Analysis.
Flanigan/Kazdan: Calculus Two: Linear and Nonlinear Functions. Second Edition.
Fleming: Functions of Several Variables. Second edition.
Foulds: Optimization Techniques: An Introduction.
Foulds: Combinatorial Optimization for Undergraduates.
Franklin: Methods of Mathematical Economics.
Halmos: Finite-Dimensional Vector Spaces. Second edition.
Halmos: Naive Set Theory.
Hämmerlin/Hoffmann: Numerical Mathematics.
Readings in Mathematics.
Iooss/Joseph: Elementary Stability and Bifurcation Theory. Second edition.
James: Topological and Uniform Spaces.
Jänich: Topology.
Kemeny/Snell: Finite Markov Chains.
Klambauer: Aspects of Calculus.
Lang: A First Course in Calculus. Fifth edition.
Lang: Calculus of Several Variables. Third edition.
Lang: Introduction to Linear Algebra. Second editon.
Lang: Linear Algebra. Third edition.
Lang: Undergraduate Algebra. Second edition.
Lang: Undergraduate Analysis.
Lax/Burstein/Lax: Calculus with Applications and Computing. Volume 1.
LeCuyer: College Mathematics with APL.
Lidl/Pilz: Applied Abstract Algebra.
Macki/Strauss: Introduction to Optimal Control Theory.
Malitz: Introduction to Mathematical Logic.
Marsden/Weinstein: Calculus I, II, III. Second edition.

(continued after Index)

Judith N. Cederberg

A Course in Modern Geometries

Springer Science+Business Media, LLC

Judith N. Cederberg
Department of Mathematics,
St. Olaf College,
Northfield, MN 55057
USA.

J.H. Ewing
Department of Mathematics,
Indiana University,
Bloomington, IN 47401,
USA.

F.W. Gehring
Department of Mathematics,
University of Michigan,
Ann Arbor, MI 48109
USA.

P.R. Halmos
Department of Mathematics,
Santa Clara University,
Santa Clara, CA 95053,
USA.

Mathematics Subject Classification (1980): 51XX

Library of Congress Cataloging-in-Publication Data

Cederberg, Judith N.
A course in modern geometries / by Judith N. Cederberg.
p. cm.
Bibliography: p.
Includes index.
1. Geometry. I. Title.
QA445.C36 1989
516—dc19

88-35478
CIP

Typeset by Thomson Press, New Delhi, India

9 8 7 6 5 4 3 2 (Corrected Second Printing, 1991)

ISBN 978-1-4757-3833-9 ISBN 978-1-4757-3831-5 (eBook)
DOI 10.1007/978-1-4757-3831-5

To Jim,
Anna, and Rachel

Preface

The origins of geometry are lost in the mists of ancient history, but geometry was already the preeminent area of Greek mathematics over 20 centuries ago. As such, it became the primary subject of Euclid's *Elements*. *Elements* was the first major example of a formal axiomatic system and became a model for mathematical reasoning. However, the eventual discoveries of non-Euclidean geometries profoundly affected both mathematical and philosophical understanding of the nature of mathematics. The relation between Euclidean and non-Euclidean geometries became apparent with the development of projective geometry—a geometry with origins in artists' questions about perspective.

This interesting historical background and the major philosophical questions raised by developments in geometry are virtually unknown to current students, who often view geometry as a dead subject full of two-column proofs of patently clear results. It is no surprise that Mary Kantowski, in an article entitled "Impact of Computing on Geometry," has called geometry "the most troubled and controversial topic in school mathematics today" (Fey, 1984, p. 31). However, this and many other recent articles provide evidence for an increasing realization that the concepts and methods of geometry are becoming more important than ever in this age of computer graphics. The geometry of the artists, projective geometry, has become the tool of computer scientists and engineers as they work on the frontiers of CAD/CAM (computer-aided design/computer-aided manufacturing) technology.

The major emphasis of this text is on the geometries developed after Euclid's *Elements* (circa 300 B.C.). In addition to the primary goal of studying these "newer" geometries, this study provides an excellent opportunity to explore aspects of the history of mathematics. Also, since algebraic techniques are frequently used, this study demonstrates the interaction of several areas of mathematics and serves to develop geometrical insights into mathematical results that previously appeared to be completely abstract in nature.

Since Euclid's geometry is historically the first major example of an axiomatic system and since one of the major goals of teaching geometry in high school is to expose students to deductive reasoning, Chapter 1 begins with a general description of axiomatic (or deductive) systems. Following this general introduction, several finite geometries are presented as examples of specific systems. These finite geometries not only demonstrate some of the concepts that occur in the geometries of Chapters 2 through 4 but also indicate the breadth of geometrical study.

In Chapter 2, Euclid's geometry is first covered in order to provide historical and mathematical preparation for the major topic of non-Euclidean geometries. This brief exposure to Euclid's system serves both to recall familiar results of Euclidean geometry and to show how few substantial changes have occurred in Euclidean geometry since Euclid formulated it. The non-Euclidean geometries are then introduced to demonstrate that these geometries, which appear similar to Euclidean geometry, have properties that are radically different from comparable Euclidean properties.

The beginning of Chapter 3 serves as a transition from the synthetic approach of the previous chapters to the analytic treatment contained in the remainder of this chapter and the next. There follows a presentation of Klein's definition of geometry, which emphasizes geometrical transformations. The subsequent study of the transformations of the Euclidean plane begins with isometries and similarities and progresses to the more general transformations called affinities.

By using an axiomatic approach and generalizing the transformations of the Euclidean plane, Chapter 4 offers an introduction to projective geometry and demonstrates that this geometry provides a general framework within which the geometries of Chapters 2 and 3 can be placed.

Although the text ends here, mathematically the next logical step in this process is the study of topology, which is usually covered in a separate course.

This text is designed for college-level survey courses in geometry. Many of the students in these courses are planning to pursue secondary-school teaching. However, with the renewed interest in geometry, other students interested in further work in mathematics or computer science will find the background provided by these courses increasingly valuable. These survey courses can also serve as an excellent vehicle for demonstrating the relationships between mathematics and other liberal arts disciplines. In an attempt to encourage student reading that further explores these relationships, each chapter begins with a section that lists suggested bibliographic sources for relevant topics in art, history, applications, and so on. I have found that having groups of students research and report on these topics not only introduces them to the wealth of expository writing in mathematics but also provides a way to share their acquired insights into the liberal arts nature of mathematics.

The material contained in this text is most appropriate for junior or senior mathematics majors. The only geometric prerequisite is some familiarity with the most elementary high-school geometry. Since the text makes frequent use of matrix algebra and occasional references to more general concepts of linear algebra, a background in elementary linear algebra is helpful. Because the text introduces the concept of a group and explores properties of geometric transformations, a course based on this text provides excellent preparation for the standard undergraduate course in abstract algebra.

I am especially grateful for the patient support of my husband and the general encouragement of my colleagues in the St. Olaf Mathematics Department. In particular, I wish to thank our department chair, Theodore Vessey, for his support and our secretary, Donna Brakke, for her assistance. I am indebted to the many St. Olaf alumni of Math 80 who studied from early drafts of the text and to Charles M. Lindsay for his encouragement after using preliminary versions of the text in his courses at Coe College in Cedar Rapids, Iowa. Others who used a preliminary version of the text and made helpful suggestions are Thomas Q. Sibley of St. John's University in Collegeville, Minnesota, and Martha L. Wallace of St. Olaf College. I am also indebted to Joseph Malkevitch of York College of the City University of New York for serving as mathematical reader for the text, and to Christina Mikulak for her careful editorial work.

Judith N. Cederberg

Contents