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# Complex Dynamics

With 28 Figures



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*On the cover:* A filled-in Julia set with parabolic fixed point, attracting petals, and repelling arms.

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