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**The Evolution of Dynamics:  
Vibration Theory from  
1687 to 1742**

With 10 Illustrations



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## Preface

In this study we are concerned with Vibration Theory and the Problem of Dynamics during the half century that followed the publication of Newton's *Principia*. The relationship that existed between these subjects is obscured in retrospection for it is now almost impossible not to view (linear) Vibration Theory as linearized Dynamics. But during the half century in question a theory of Dynamics did not exist; while Vibration Theory comprised a good deal of acoustical information, posed definite problems and obtained specific results. In fact, it was through problems posed by Vibration Theory that a general theory of Dynamics was motivated and discovered.

Believing that the emergence of Dynamics is a critically important link in the history of mathematical science, we present this study with the primary goal of providing a guide to the relevant works in the aforementioned period. We try above all to make the contents of the works readily accessible and we try to make clear the historical connections among many of the pertinent ideas, especially those pertaining to Dynamics in many degrees of freedom. But along the way we discuss other ideas on emerging subjects such as Calculus, Linear Analysis, Differential Equations, Special Functions, and Elasticity Theory, with which Vibration Theory is deeply interwound. Many of these ideas are elementary but they appear in a surprising context: For example the eigenvalue problem does not arise in the context of special solutions to linear problems—it appears as a condition for isochronous vibrations.

Although mathematical thought differs in different ages, mathematics itself has a coherence that transcends time. Thus it provides a powerful tool with which to grasp modes of thought from former times. From an immersion in the details of mathematical arguments, one can gather enough precise understanding to be able to enter into the domain of the intuitive. Therefore we believe that our study not only describes a link in the evolution of a specific subject but also that it assists in the attainment of a feel for physics in the age of Newton and the Bernoullis.

In spite of its evident importance, dynamics in the first half of the eighteenth century has been largely neglected. This is the period of late Newton and early Euler; thus it lies in the shadow of great brilliance coming from both before and after. For example, Euler was a central figure; but

his works from the period go with little notice because he later reworked everything in a form and from a point of view that have become generally familiar. Thus Truesdell's notes on Euler were pioneering works.<sup>1</sup> Truesdell emphasized the fact that the idea of dynamical equations was slow to emerge; furthermore, he provided a basic indication of the contents of a vast number of papers, including most of the papers considered in the present study. We gratefully acknowledge our indebtedness to his notes.

Yellow Springs  
April 1981

J.C. and S.D.

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<sup>1</sup> Truesdell [1, 2].

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