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Francesco Calogero

Classical Many-Body Problems Amenable to Exact Treatments

(Solvable and/or Integrable and/or Linearizable...) in One-, Two- and Three-Dimensional Space

Springer

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Foreword

This book focuses on *exactly treatable* many-body problems. This class does not include most physical problems. We are therefore reminded "of the story of the man who, returning home late at night after an alcoholic evening, was scanning the ground for his key under a lamppost; he knew, to be sure, that he had dropped it somewhere else, but only under the lamppost was there enough light to conduct a proper search" <C71>. Yet we feel the interest for such models is nowadays sufficiently widespread – because of their beauty, their mathematical relevance and their multifarious applicative potential – that no apologies need be made for our choice. In any case, whoever undertakes to read this book will know from its title what she is in for!

Yet this title may require some explanations: a gloss of it (including its extended version, see inside front cover) follows.

By "Classical" we mean *nonquantal* and *nonrelativistic* (although some consider the Ruijsenaars-Schneider models, which are indeed treated in this book, as relativistic versions of, previously known, nonrelativistic models; see below): our presentation is mainly focussed on many-body systems of point particles whose time evolution is determined by Newtonian equations of motion (*acceleration proportional to force*). The fact that we treat problems not only in one, but also in two, and even in three (and occasionally in an arbitrary number of), dimensions, is of course somewhat of a novelty: indeed the treatment of two-dimensional, and especially three-dimensional, (rotation-invariant!) models, is based on recent (sometimes very recent) findings. By "amenable to exact treatments" we mean that, to investigate the behavior of the many-body models identified and studied in this book, significant progress can be made by "exact" (i. e., not approximate) techniques. The extent to which one can thereby master the detailed behavior of these many-body systems varies from case to case: this is emphasized by the parenthetical part of our title, which perhaps requires some additional elaboration, to explain what we mean by our distinction – which is, to be sure, a *heuristic* one: quite useful, but not quite precise – among *solvable, integrable* and *linearizable* models.

Solvable models are characterized by the availability of a technique of solution which requires purely *algebraic operations* (such as inverting or diagonalizing finite matrices, or finding the zeros of known polynomials), and/or possibly solving *known* (generally linear, possibly nonautonomous) ODEs in terms of *known* special functions (say, of hypergeometric type), and/or perhaps the *inversion* of known functions (as in the standard *solution by quadratures*).

Integrable models are those for which some approach (for instance, a "Lax-pair", see below) is available, which yields an adequate supply of *constants of motion.* As a rule these models are also solvable, but generally this requires more labor. In the Hamiltonian cases, these models are generally *Liouville integrable.*

Thirdly, we refer to *linearizable* problems: their treatment generally requires, in addition to the operations mentioned above in the context of *solvable* models, the solution of *linear, generally nonautonomous, ODEs,* which, in spite of their being generally rather simple, might indeed give rise to quite complicated (chaotic?) motions. In the Hamiltonian cases these many-body models need not be integrable in the Liouville sense, although the linearity of the equations to be finally solved entails the possibility to introduce *constants of the motion* via the *superposition principle,* which guarantees that the general solution of a linear ODE can be represented as a linear combination with *constant* coefficients of an appropriate set of specific solutions. In any case a linearizable many-body problem is certainly much easier to treat than the generic (nonlinear!) many-body problem, inasmuch as its solution can be reduced to solving a *linear* first-order matrix ODE (indeed, in most cases, a *single linear second-order scalar* ODE – albeit a *nonautonomous* one – see below).

Clearly these three categories of problems – *solvable, integrable, linearizable* – are ordered in terms of increasing difficulty, so that (as indeed the title of this book indicates with its *and/or*'s), problems belonging to a lower category generally also belong to the following one(s).

But let us reemphasize that the distinction among *solvable, integrable* and *linearizable* models is imprecise: the boundaries among these categories are somewhat blurred, moreover we have been vague about what "solving" a problem really means: Finding the general solution? Solving the initial-value problem? For which class of initial data? And what about boundary conditions (which in some cases are essential to define the problem)? The final dots in the title underline the heuristic, and incomplete, character of this distinction among *solvable, integrable* and *linearizable* models (for instance, we shall also introduce below the notion of *partially solvable* models, whose initial-value problem can be solved only for a restricted subclass of initial data). Yet this distinction is convenient to convey synthetically the status of the various many-body problems treated in this book.

Two additional remarks.

(i). The genesis of exactly treatable models comes seldom from the discovery of a technique to solve a given problem; generally the actual development is the other way round, a suitable technique is exploited to discover all the models which can be treated (possibly solved) by it. Some disapprove of such an approach to research, in which, rather than trying to find the solution of a problem, one tries to find problems that fit a known (technique of) solution. Some, indeed, go as far as decrying "basic research," presumably because, in contrast to applied research, it does not solve specific problems: "Basic research is like shooting an arrow into the air and, where it lands, painting a target" (attributed to Homer Adkins (1984) <APS99>). This author, on the contrary, does not see anything wrong with this approach; it seems to me it is a normal way of making progress in science. For instance: occasionally an experimental device (say, a particle accelerator) is built for the specific purpose to discover something (say, a new elementary particle); but more often an experimental device is available (say, a particle accelerator), and the experimental activity is concentrated on whatever that particular device allows experimenters to do. And nobody sees anything wrong in this. Indeed there is a quotation from Carl Jacobi (which I am lifting from a classical treatise by Vladimir Arnold <A74>), that expresses this point of view in a context quite close to that of this book (although it refers specifically to an approach -- separation of variables -- we do not explicitly treat): "The main difficulty to integrate these differential equations is to find the appropriate change of variables. There is no rule to discover it. Hence we need to follow the inverse path, namely to introduce some convenient change of variables and investigate to which problems it can be successfully applied." And another quotation which expresses a point of view I sympathize with comes from Vladimir E. Zakharov: "A mathematician, using the dressing method to find a new integrable system, could be compared with a fisherman, plunging his net into the sea. He does not know what a fish he will pull out. He hopes to catch a goldfish, of course. But too often his catch is something that could not be used for any known to him purpose. He invents more and more sophisticated nets and equipments and plunges all that deeper and deeper. As a result he pulls on the shore after a hard work more and more strange creatures. He should not despair, nevertheless. The strange creatures may be interesting enough if you are not too pragmatic. And who knows how deep in the sea do goldfishes live? ". <Z90>

(ii). Models amenable to exact treatments are, of course, special. Why focus on them, rather than look at general cases, which capture many more problems, including the more "physical" ones? But again, this is to a large extent the essence of normal science. Pythagora' s theorem does not hold for all triangles, but only for rectangular ones. Should this be considered a shortcoming of this mathematical result, or instead its very essence? The answer is plain.

Finally, a few remarks on the presentation and the selection of the material.

The presentation is meant to facilitate the self-education of a reader who wishes to enter this research area. For instance, special cases are often presented in place or in advance of more general treatments, in order to introduce ideas and techniques in a simpler context. The division into a main text and a secondary part, separated by horizontal lines and distinguished by a slight difference in the size of the fonts, should also be helpful: in the secondary part we generally segregate remarks and arguments (often including proofs) which deviate from the main flow of the presentation (but the reader is well advised to read sequentially through these parts as well, which often contain material that is essential -- or at least helpful -- for the understanding of what follows; and this advise also applies to all *exercises*, which should all be read, even when there is no intention/possibility to invest immediately time in their solution). Almost all mentions of related references, historical remarks, due credits, etc., are also relegated elsewhere, to special sections ("Notes") located at the end of the chapters and of some appendices. Of course this book might also be used as background material for teaching a course (it actually emerged from such a context – indeed, it profited from such a test) .

The selection of the material presented in this book is unashamedly skewed towards research topics to which the author has personally contributed, or which he finds particularly congenial (such as the Ruijsenaars-Schneider model). The enormous amount of research on the topics treated in this book and/or on closely related areas that emerged in the last quarter century would have anyway doomed to failure any effort at providing a "complete" coverage; likewise any attempt to present a "complete" bibliographic record of the contributions on the topics treated would have been impossible, indeed perhaps futile given the great ease nowadays to retrieve relevant references via computer-assisted searches. These are admittedly lame excuses for the shortcomings of this book, whose worth (be it somewhat positive or largely negative) will in any case be best assessed by those who will use it as a (personal or didactic) teaching tool; but I like to express here my apologies to all those colleagues who contributed importantly to the development of this area of research and who will not find in this book any reference to their contribution.

The organization of the book into a rather detailed net of telescoped sections is meant to help the reader, both the first time he navigates through the book as well as when she might wish to retrieve some notion. Moreover, the table of contents provides a synthetic overview of the material covered in this book which might help the perplexed browser in deciding whether he wishes to become an engaged, or even a diligent, reader. Equations are numbered progressively within each section and appendix: equation (16) of Section 2.1.1 is referred to as (16) within that section, as (2.1.1.-16) elsewhere; and likewise (C.-10b) is equation (10b) of Appendix C (but within Appendix C it is referred to simply as (10b)).

Let me end this Foreword on a personal note. My father, Guido Calogero, was a philosopher who wrote many books (without formulas!), and he also had a great interest for, and much scholarship in, philology and archaeology (especially texts from ancient Greece). Hence, he always paid a keen attention to the appearance of any text; and he much disliked misprints. I inherited this attitude, but not his keen eye to weed out imperfections. Hence I must apologize for the many misprints and other defects this book certainly contains, and beg the reader to take the same benevolent attitude displayed by Hermann Weyl in his 1938 review <W38> of the second volume of the classic mathematical physics treatise by Richard Courant and David Hilbert <CH37>, when he wrote: "The author apologizes that lack of time prevented him from fitting out this book with a full sized index of literature and such paraphernalia. The same reason may be responsible for quite a few misprints on which the reader will occasionally stumble. But perhaps even these minor faults deserve praise rather than blame. Although I know that a craftsman's pride should be in having his work as perfect and shipshape as possible, even in the most minute and inessential details, I sometimes wonder whether we do not lavish on the dressing-up of a book too much time that would better go into more important things."

Yet I will be most grateful to whoever will take the trouble to bring to my attention shortcomings of this book (including misprints!), via an e-mail message sent to (both) these addresses: francesco.calogero@uniroma1.it, francesco.calogero@roma1.infn.it.

Preface

This book, as well as its title, are long, perhaps too long; and it took quite a long time to complete this project, well over three years of intense hard work. Throughout this period I sought and got advise from several colleagues and friends, and also from students to whom preliminary drafts were distributed and who helped me by spotting misprints and mistakes (letting them search for these turned indeed out to be a very efficient teaching technique!). For a special word of thanks I like to mention Mario Bruschi, Jean-Pierre Françoise, David Gomez Ullarte, Misha Olshanetsky, Orlando Ragnisco, Simon Ruijsenaars. But it is of course understood that I am solely responsible for all shortcomings of this book.

I also wish to thank: Alessandra Grussu and Matteo Sommacal for transforming my scribbled first draft into WORD files for me to work on; my Physics Department at the University of Rome I "La Sapienza" for supporting financially this typing job, and in particular the Administrator of my Department, Maria Vittoria Marchet, for organizing this arrangement, and the Director of my Department, Francesco Guerra, for encouraging me to undertake this project; and the staff at Springer, in particular Mrs. Brigitte Reichel-Mayer respectively Prof. Wolf Beiglboeck, for their cooperative attitude on the technical respectively substantive aspects of the production of this book.

This book is dedicated to the memory of Juergen Moser, whose seminal work was instrumental in opening up this field of research. Most regrettably, I never managed to meet him: I only spoke by telephone with him one time, more than twenty years ago, from JFK airport in New York, while he was in his office at the Courant Institute; then, through the years, various last minutes glitches postponed more than once our getting together. Alas, now it is too late to remedy this mistake.

September 2000 *Francesco Calogero*

Contents