

Peter Bürgisser

Completeness and Reduction in Algebraic Complexity Theory

With 16 Figures



Springer

Peter Bürgisser
Fachbereich 17 · Mathematik-Informatik
Universität-Gesamthochschule Paderborn
Warburger Strasse 100
33095 Paderborn, Germany
e-mail: pbuerg@math.uni-paderborn.de

Library of Congress Cataloging-in-Publication Data

Bürgisser, Peter, 1962-. Completeness and reduction in algebraic complexity theory / Peter Bürgisser. p. cm. – (Algorithms and computation in mathematics, ISSN 1431-1550; v. 7)
Includes bibliographical references and index.

1. Computational complexity. I. Title. II. Series QA267.7.B88 2000 511.3-dc21 00-029647

Mathematics Subject Classification (2000): 68Q05, 68Q15, 68Q25, 68Q40,
15A15, 22E70, 33C25, 03D15, 03D25, 05Cxx, 82B20

ISSN 1431-1550

ISBN 978-3-642-08604-5 ISBN 978-3-662-04179-6 (eBook)

DOI 10.1007/978-3-662-04179-6

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 2000

Originally published by Springer-Verlag Berlin Heidelberg New York in 2000.

Softcover reprint of the hardcover 1st edition 2000

Cover design: MetaDesign plus GmbH, Berlin

Cover production; *design & production* GmbH, Heidelberg

Typeset in L^AT_EX by the author

Printed on acid-free paper SPIN 10743448 46/3143Ko – 5 4 3 2 1 0

Contents

1	Introduction	1
1.1	Classical Model	1
1.2	Blum-Shub-Smale Model	3
1.3	Valiant's Model	4
1.4	Overview of Main Results	6
2	Valiant's Algebraic Model of NP-Completeness	11
2.1	The Complexity Classes VP and VNP	11
2.2	Completeness of the Permanent Family	16
2.2.1	p-Definability and Formula Size	16
2.2.2	Universality of the Permanent	19
2.2.3	Completeness of the Permanent	21
2.3	Closure Properties	25
2.4	Parallel Complexity	30
2.5	Completeness of the Determinant Family	34
3	Some Complete Families of Polynomials	37
3.1	Generating Functions of Graph Properties	37
3.2	p-Computable Families	39
3.3	VNP-Complete Families	41
3.3.1	Matchings	42
3.3.2	Cliques	43
3.3.3	Cycle Format Polynomials	44
3.3.4	Graph Factors	48
3.3.5	Hamilton Cycles of Planar Graphs	50
3.3.6	Self Avoiding Walks	53
3.3.7	Connectivity	55
4	Cook's versus Valiant's Hypothesis	61
4.1	Dependence on the Field	61
4.2	Statement of Main Results	64
4.3	Review of Discrete Complexity Classes	66
4.4	Relating NP to Counting Classes	68
4.5	A Bound on the Heights	70
4.6	Roots of Univariate Polynomials Modulo a Prime	75
4.7	Proof of Thm. 4.5	76

5	The Structure of Valiant's Complexity Classes	81
5.1	Outline and Comparison with Previous Work	81
5.2	An Abstract Diagonalization Theorem	82
5.3	An Abstract Embedding Theorem	85
5.4	Structure of Valiant's Complexity Classes	89
5.5	A Specific Family Neither Complete Nor p-Computable	92
5.6	Relativized Complexity Classes	96
6	Fast Evaluation of Representations of General Linear Groups	105
6.1	Description of the Problem	105
6.2	Preliminaries on Representations of GL_m	107
6.3	Auxiliary Fast Linear Algebra Algorithms	109
6.4	An Algorithm for Evaluating Representations	112
6.5	A Lower Bound	114
6.6	Fast Evaluation of Legendre Functions	115
7	The Complexity of Immanants	117
7.1	Motivation and Outline of Chapter	117
7.2	Fast Evaluation of Immanants	118
7.3	Completeness Results for Immanants	120
7.4	Character Formulas for the Symmetric Group	121
7.5	p-Definability of Immanants	126
7.6	Completeness Proofs	128
8	Separation Results and Future Directions	135
8.1	Specific Families Which Are Not p-Definable	135
8.2	Separations for the Complexity Class VQP	139
8.3	Possible Connections to Univariate Polynomials	141
8.4	Connections to the BSS-Model	143
	References	149
	List of Notation	159
	Index	163