

STUDENT MATHEMATICAL LIBRARY  
Volume 8

**EXPLORING THE NUMBER JUNGLE:  
A JOURNEY INTO DIOPHANTINE ANALYSIS**

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# Table of contents

|   |    |
|---|----|
| <i>Opening thoughts: Welcome to the jungle</i>        | 1  |
| Modules   |    |
| 1. A bit of foreshadowing and some rational rationale | 7  |
| 2. Building the rationals via Farey Sequences         | 11 |
| 3. Discoveries of Dirichlet and Hurwitz               | 17 |
| 4. The theory of continued fractions                  | 23 |
| 5. Enforcing the law of best approximates             | 29 |
| 6. Markoff's spectrum and numbers                     | 35 |
| 7. Badly approximable numbers and quadratics          | 43 |
| 8. Solving the alleged "Pell" equation                | 49 |
| 9. Liouville's work on numbers algebraic and not      | 55 |
| 10. Roth's stunning result and its consequences       | 61 |
| 11. Pythagorean triples through diophantine geometry  | 67 |
| 12. A quick tour through elliptic curves              | 75 |
| 13. The geometry of numbers                           | 81 |
| 14. Simultaneous diophantine approximation            | 89 |
| 15. Using geometry to sum some squares                | 95 |
| 16. Spinning around irrationally and uniformly        | 99 |

|   |     |
|---|-----|
| 17. A whole new world of $p$ -adic numbers              | 105 |
| 18. A glimpse into $p$ -adic analysis                   | 113 |
| 19. A new twist on Newton's method                      | 117 |
| 20. The power of acting locally while thinking globally | 123 |
| Appendices  |     |
| 1. Selected Big Picture Question commentaries           | 129 |
| 2. Hints and remarks                                    | 135 |
| 3. Further reading                                      | 145 |
| Acknowledgments   | 147 |
| Index   | 149 |