

How Round Is Your Circle?

WHERE ENGINEERING AND MATHEMATICS MEET

John Bryant and Chris Sangwin

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