Graduate Texts in Mathematics 87

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Graduate Texts in Mathematics

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Cohomology of Groups

Springer

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Library of Congress Cataloging in Publication Data
Brown, Kenneth S.
Cohomology of groups.
(Graduate texts in mathematics; 87)
Bibliography: p.
Includes index.
1. Groups, Theory of. 2. Homology theory.
I. Title. II. Series.
QA171.B876 512'.22 82-733
AACR2

ISBN 978-1-4684-9329-0 ISBN 978-1-4684-9327-6 (eBook)
DOI 10.1007/978-1-4684-9327-6

Preface

This book is based on a course given at Cornell University and intended primarily for second-year graduate students. The purpose of the course was to introduce students who knew a little algebra and topology to a subject in which there is a very rich interplay between the two. Thus I take neither a purely algebraic nor a purely topological approach, but rather I use both algebraic and topological techniques as they seem appropriate.

The first six chapters contain what I consider to be the basics of the subject. The remaining four chapters are somewhat more specialized and reflect my own research interests. For the most part, the only prerequisites for reading the book are the elements of algebra (groups, rings, and modules, including tensor products over non-commutative rings) and the elements of algebraic topology (fundamental group, covering spaces, simplicial and CW-complexes, and homology). There are, however, a few theorems, especially in the later chapters, whose proofs use slightly more topology (such as the Hurewicz theorem or Poincaré duality). The reader who does not have the required background in topology can simply take these theorems on faith.

There are a number of exercises, some of which contain results which are referred to in the text. A few of the exercises are marked with an asterisk to warn the reader that they are more difficult than the others or that they require more background.

I am very grateful to R. Bieri, J-P. Serre, U. Stammbach, R. Strebel, and C. T. C. Wall for helpful comments on a preliminary version of this book.

Notational Conventions

All rings (including graded rings) are assumed to be associative and to have an identity. The latter is required to be preserved by ring homomorphisms. Modules are understood to be *left* modules, unless the contrary is explicitly stated. Similarly, group actions are generally understood to be left actions.

If a group G acts on a set X, I will usually write X/G instead of $G\backslash X$ for the orbit set, even if G is acting on the left. One exception to this concerns the notation for the set of cosets of a subgroup H in a group G. Here we are talking about the left or right translation action of H on G, and I will always be careful to put the H on the appropriate side. Thus $G/H = \{gH : g \in G\}$ and $H\backslash G = \{Hg : g \in G\}$.

A symbol such as

$$\sum_{g \in G/H} f(g)$$

indicates that f is a function on G such that $f(g)$ depends only on the coset gH of g; the sum is then taken over a set of coset representatives.

Finally, I use the "topologists' notation"

$$\mathbb{Z}_n = \mathbb{Z}/n\mathbb{Z};$$

in particular, $\mathbb{Z}_p$ denotes the integers mod p, not the p-adic integers.

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