

Alexander M. Bronstein
Michael M. Bronstein
Ron Kimmel

Numerical Geometry of Non-Rigid Shapes

With 10 Color Figures

 Springer

Alexander M. Bronstein
Technion-Israel Institute of Technology
Haifa, Israel
bron@cs.technion.ac.il

Michael M. Bronstein
Technion-Israel Institute of Technology
Haifa, Israel
mbron@cs.technion.ac.il

Ron Kimmel
Technion-Israel Institute of Technology
Haifa, Israel
ron@cs.technion.ac.il

ISSN: 0172-603x

ISBN: 978-0-387-73300-5

e-ISBN: 978-0-387-73301-2

DOI 10.1007/978-0-387-73301-2

Library of Congress Control Number: 2008934481

Mathematics Subject Classification (2000): 53A05, 52C25, 49M37

© 2008 Springer Science+Business Media, LLC

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC, 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Printed on acid-free paper

springer.com

Contents

Foreword	VII
About the Authors	IX
Preface	XI
1 Introduction	1
1.1 Similarity of non-rigid shapes	3
1.2 Correspondence problems	6
1.3 A landscape of problems	7
Notes	9
2 A Taste of Geometry	11
2.1 Basic terms in metric geometry and topology	11
2.2 Isometries	13
2.3 Length spaces	17
2.4 Manifolds	20
2.5 Embedded surfaces	21
2.6 Curvature and the second fundamental form	26
2.7 Intrinsic view on geometry of surfaces	29
2.8 Bending and rigidity	31
2.9 Intrinsic invariants	34
Suggested reading	37
Problems	37
Notes	39

3	Discrete Geometry	41
3.1	Point clouds and sampling	41
3.2	Farthest point sampling	43
3.3	Voronoi tessellation	46
3.4	Centroidal Voronoi sampling and the Lloyd-Max algorithm ...	48
3.5	Connectivity	52
3.6	Delaunay tessellation	53
3.7	Triangular meshes	54
3.8*	Local feature size and curvature-dependent sampling	57
3.9*	Approximation quality	61
	Suggested reading	63
	Software	63
	Problems	63
	Notes	64
4	Shortest Paths and Fast Marching Methods	67
4.1	The shortest path problem	67
4.2	Dijkstra's shortest path algorithm	69
4.3	Fast marching methods	71
4.4	Fast marching on parametric surfaces	81
4.5	Marching even faster	83
4.6	Parallel distance computation	85
4.7*	Minimal geodesics	87
	Suggested reading	89
	Software	90
	Problems	90
	Notes	91
5	Numerical Optimization	93
5.1	Local versus global optimization	93
5.2	Optimality conditions	94
5.3	Unconstrained optimization algorithms	97
5.4	The quest for a descent direction	100
5.5	Preconditioning	104
5.6	Let Newton be!	105
5.7	Truncated Newton	106
5.8	Quasi-Newton algorithms	107
5.9	Non-convex optimization	108
5.10	Constrained optimization	110
5.11	Penalty and barrier methods	112
5.12*	Augmented Lagrangian method	114
	Suggested reading	116
	Software	116
	Problems	116
	Notes	118

6	In the Rigid Kingdom	119
	6.1 Moments of joy, moments of sorrow	120
	6.2 Iterative closest point algorithms	125
	6.3 Enter numerical optimization	128
	6.4 Rigid correspondence	131
	Suggested reading	133
	Software	133
	Problems	133
	Notes	134
7	Multidimensional Scaling	137
	7.1 Isometric embedding problem	138
	7.2 Multidimensional scaling	142
	7.3 SMACOF algorithm	143
	7.4* Second-order methods	146
	7.5 Variations on the stress theme	148
	7.6 Multiresolution methods	153
	7.7* Multigrid MDS	156
	7.8* Vector extrapolation	160
	7.9 A trouble with topology	164
	Suggested reading	165
	Software	166
	Problems	167
	Notes	167
8	Spectral Embedding	169
	8.1 Classic MDS	170
	8.2 Local methods	173
	8.3 The Laplace-Beltrami operator	176
	8.4 To hear the shape of the drum	178
	8.5* Discrete Laplace-Beltrami operator	180
	Suggested reading	184
	Software	184
	Problems	184
	Notes	185
9	Non-Euclidean Embedding	187
	9.1 Spherical embedding	187
	9.2 Generalized multidimensional scaling	192
	9.3 Representation issues	194
	9.4 Geodesic distance computation	197
	9.5 Minimization of the generalized stress	198
	9.6 Multiresolution encore	202
	Suggested reading	203
	Software	203

Problems	204
Notes	204
10 Isometry-Invariant Similarity	205
10.1 Equivalence, similarity, and distance	205
10.2 Embedding distance	207
10.3 Gromov-Hausdorff distance	208
10.4 Intrinsic symmetry	211
Suggested reading	214
Problems	214
11 Partial Similarity	217
11.1 Recognition by parts	218
11.2 Paretian approach to partial similarity	221
11.3 Scalar partial similarity	224
11.4 Fuzzy approximation	226
11.5 Extrinsic partial similarity	229
11.6 Intrinsic partial similarity	230
11.7 Not only size matters	232
Suggested reading	236
Problems	236
Notes	238
12 Non-rigid Correspondence and Calculus of Shapes	239
12.1 Intrinsic parameterization	240
12.2 An image processing approach	241
12.3 Minimum distortion correspondence	244
12.4 Texture mapping and transfer	246
12.5 Morphing	249
12.6*Guaranteed self-intersection free morph	254
12.7 Calculus of shapes	255
Suggested reading	258
Software	258
Problems	259
Notes	259
13 Three-dimensional Face Recognition	261
13.1 Some terminology	263
13.2 A retrospective	264
13.3 Isometric model of facial expressions	268
13.4 Expression-invariant face recognition	269
13.5 Comparison of photometric properties	273
Suggested reading	275
Notes	275

14 Epilogue 277

Solutions of Selected Problems 279

Software 293

Notation 297

Acronyms 299

Glossary 301

References 307

Subject Index 327

Author Index 335