

# MEMOIRS

of the  
American Mathematical Society

---

Number 663

---

## Iterated Function Systems and Permutation Representations of the Cuntz Algebra

Ola Bratteli  
Palle E. T. Jorgensen



---

May 1999 • Volume 139 • Number 663 (second of 5 numbers) • ISSN 0065-9266

---

**American Mathematical Society**  
Providence, Rhode Island

# Contents

List of Figures	ix
Chapter 1. Introduction	1
Chapter 2. Permutative representations of $\mathcal{O}_N$	7
Chapter 3. Monomial representations and integers modulo $N$	14
Chapter 4. Cycles of irreducible $\mathcal{O}_N$ representations and their atoms of $\text{UHF}_N$ representations	26
Chapter 5. Relations of finite type and sub-Cuntz states	28
Chapter 6. The shift representation	31
Chapter 7. The universal permutative multiplicity-free representation	34
Chapter 8. Some specific examples of the cycle and atom structure: The mod $N$ case	36
8.1. The case $N = 2$	36
8.2. The case $N$ general, $s_i = i$ for $i \in \mathbb{Z}_N = \{0, 1, \dots, N - 1\}$	42
8.3. The case $N = 3$ , $\{s_i\} = \{1, 3, 5\}$	42
Chapter 9. Some specific examples of the cycle and atom structure: The mod $N$ case	43
9.1. A matrix case	43
9.2. Another matrix case: full spectrum	47
9.3. Self-similarity and tiles	49
9.4. Tiles in the plane	55
9.5. Single-atom cases	57
9.6. Multiple coverings	60
9.7. Box spline matrices	70
Chapter 10. The general mod $N$ situation	76
Chapter 11. Concluding remarks	83
Bibliography	86
List of Symbols	89