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Topics on Concentration Phenomena and Problems with Multiple Scales

Springer

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Preface

The research group 'Homogenization Techniques and Asymptotic Methods for Problems with Multiple Scales', co-ordinated by Valeria Chiadò Piat and funded by INdAM-GNAMPA (*Istituto Nazionale di Alta Matematica-Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni*), operated from 2001 to 2005, involving in its activities a number of young Italian mathematicians, mainly interested in problems in the Calculus of Variations and Partial Differential Equations. One of the initiatives of that group has been the organization of a number of schools. Those in the years 2001-2003, whose lecture notes are gathered in this book, had been devoted to problems with oscillations and concentrations, while the schools in the years 2004-2005 covered a range of topics of Applied Mathematics.

The first school in Turin, 17–21 September 2001, bearing the name of the research group and devoted to problems with multiple scales, was partially disrupted by the events of September 11 of that year, one speaker, Gilles Francfort, finding himself grounded in Los Angeles, and two other speakers, Andrey Piatnitski and Gregory Chechkin, slowed down in their car trip to Italy by the tightening of the borders around the European Community. The two remaining speakers managed however to enlarge their courses to cover some extra material, encouraged by the receptive audience. The course of Andrea Braides was devoted to the description of the behaviour of variational problems on lattices as the lattice spacing tends to zero, and the various multi-scale behaviours that may be obtained from this process; that of Anneliese Defranceschi to energies with competing bulk and surface interactions. The extra lectures are not included in these notes, but some of them constitute part of the material in the book 'Γ-convergence for Beginners' (Oxford U.P., 2002) by Braides. The course of Francfort, on H-measures, was later recovered in a 'Part II' of the School held at IAC in Rome, December 3–5, 2001, together with a contribution of Roberto Peirone on homogenization on fractals. Here we also include the text of the two courses of Piatnitski and Chechkin, while the lecture notes of the course by Francfort have appeared as a chapter of the book 'Variational Problems in Materials Science', Birkhäuser, 2006.

The second part of the present notes covers the content of the subsequent school on 'Concentration Phenomena for Variational Problems' held at the Department of Mathematics of the University of Rome 'La Sapienza', September 1–5, 2003 (co-organized by A. Braides, which explains why he appears both as an editor and as a contributor). Scope of the School was to present different problems in the Calculus of Variations depending on a small parameter ε, that exhibit a dramatic 'change of type' as this parameter tends to 0, that is best described by the 'concentration' of some quantity at some lower-dimensional set. The courses of Sylvia Serfaty and Didier Smets treat the case of Ginzburg-Landau energies. In a two-dimensional setting it is known that the concentration of Jacobians of minimizers at points can be interpreted as the arising of 'vortices'. A novel method envisaged by Sandier and Serfaty shows how the limit motion of these vortices can be described by making use of Γ-convergence. On the other hand, Smets's course focuses on the information that can be obtained by looking at the fine behaviour of solutions of the Allen-Cahn equations, and concerns the motion in any dimension. The use of Γ-convergence as a way to describe the concentration of maximizers of problems with sub-critical growth is also the subject of the third course by Adriana Garroni. Here the concentrating quantity is not a Jacobian (the problem is scalar), but a suitable scaling of the square of the gradient of the maximizer, that converges as a measure to a sum of Dirac masses. This phenomenon has been previously described by means of the Concentration-Compactness alternative, and this 'version' by Γ-convergence gives a new interpretation of the results.

The course of Sylvia Serfaty, originally programmed for this school, had to be postponed to a subsequent spin-off 'School on Geometric Evolution Problems' held at the Department of Mathematics of the University of Rome 'Tor Vergata', January 26–28, 2004 (with the same organizing team, and an additional course by Giovanni Bellettini) but is considered essentially part of the September 2003 School, and that is why it is included here. Other two courses, whose notes are not presented here, were held at the School by Giovanni Alberti and Halil Mete Soner. The course of Soner followed the notes of a previous school and can be found in his Lecture Notes 'Variational and dynamic problems for the Ginzburg-Landau functional. Mathematical aspects of evolving interfaces' (*Lecture Notes in Math.* **1812**, Springer, 2003, 177–233). Alberti's presentation is partly covered by his review paper 'A variational convergence result for Ginzburg-Landau functionals in any dimension' (*Boll. Un. Mat Ital.* **4** (2001), 289–310). As a final acknowledgement, it must be mentioned that these schools had been additionally jointly sponsored by the Rome and Milan Units of the National Project 'Calculus of Variations'.

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