

Contents

Preface	xi
1 Introduction	1
1.1 Mathematical optimization	1
1.2 Least-squares and linear programming	4
1.3 Convex optimization	7
1.4 Nonlinear optimization	9
1.5 Outline	11
1.6 Notation	14
Bibliography	16
I Theory	19
2 Convex sets	21
2.1 Affine and convex sets	21
2.2 Some important examples	27
2.3 Operations that preserve convexity	35
2.4 Generalized inequalities	43
2.5 Separating and supporting hyperplanes	46
2.6 Dual cones and generalized inequalities	51
Bibliography	59
Exercises	60
3 Convex functions	67
3.1 Basic properties and examples	67
3.2 Operations that preserve convexity	79
3.3 The conjugate function	90
3.4 Quasiconvex functions	95
3.5 Log-concave and log-convex functions	104
3.6 Convexity with respect to generalized inequalities	108
Bibliography	112
Exercises	113

4	Convex optimization problems	127
4.1	Optimization problems	127
4.2	Convex optimization	136
4.3	Linear optimization problems	146
4.4	Quadratic optimization problems	152
4.5	Geometric programming	160
4.6	Generalized inequality constraints	167
4.7	Vector optimization	174
	Bibliography	188
	Exercises	189
5	Duality	215
5.1	The Lagrange dual function	215
5.2	The Lagrange dual problem	223
5.3	Geometric interpretation	232
5.4	Saddle-point interpretation	237
5.5	Optimality conditions	241
5.6	Perturbation and sensitivity analysis	249
5.7	Examples	253
5.8	Theorems of alternatives	258
5.9	Generalized inequalities	264
	Bibliography	272
	Exercises	273
II	Applications	289
6	Approximation and fitting	291
6.1	Norm approximation	291
6.2	Least-norm problems	302
6.3	Regularized approximation	305
6.4	Robust approximation	318
6.5	Function fitting and interpolation	324
	Bibliography	343
	Exercises	344
7	Statistical estimation	351
7.1	Parametric distribution estimation	351
7.2	Nonparametric distribution estimation	359
7.3	Optimal detector design and hypothesis testing	364
7.4	Chebyshev and Chernoff bounds	374
7.5	Experiment design	384
	Bibliography	392
	Exercises	393

Contents

ix

8 Geometric problems	397
8.1 Projection on a set	397
8.2 Distance between sets	402
8.3 Euclidean distance and angle problems	405
8.4 Extremal volume ellipsoids	410
8.5 Centering	416
8.6 Classification	422
8.7 Placement and location	432
8.8 Floor planning	438
Bibliography	446
Exercises	447
III Algorithms	455
9 Unconstrained minimization	457
9.1 Unconstrained minimization problems	457
9.2 Descent methods	463
9.3 Gradient descent method	466
9.4 Steepest descent method	475
9.5 Newton's method	484
9.6 Self-concordance	496
9.7 Implementation	508
Bibliography	513
Exercises	514
10 Equality constrained minimization	521
10.1 Equality constrained minimization problems	521
10.2 Newton's method with equality constraints	525
10.3 Infeasible start Newton method	531
10.4 Implementation	542
Bibliography	556
Exercises	557
11 Interior-point methods	561
11.1 Inequality constrained minimization problems	561
11.2 Logarithmic barrier function and central path	562
11.3 The barrier method	568
11.4 Feasibility and phase I methods	579
11.5 Complexity analysis via self-concordance	585
11.6 Problems with generalized inequalities	596
11.7 Primal-dual interior-point methods	609
11.8 Implementation	615
Bibliography	621
Exercises	623

Appendices	631
A Mathematical background	633
A.1 Norms	633
A.2 Analysis	637
A.3 Functions	639
A.4 Derivatives	640
A.5 Linear algebra	645
Bibliography	652
B Problems involving two quadratic functions	653
B.1 Single constraint quadratic optimization	653
B.2 The S-procedure	655
B.3 The field of values of two symmetric matrices	656
B.4 Proofs of the strong duality results	657
Bibliography	659
C Numerical linear algebra background	661
C.1 Matrix structure and algorithm complexity	661
C.2 Solving linear equations with factored matrices	664
C.3 LU, Cholesky, and LDL ^T factorization	668
C.4 Block elimination and Schur complements	672
C.5 Solving underdetermined linear equations	681
Bibliography	684
References	685
Notation	697
Index	701