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Differential Forms in Algebraic Topology

With 92 Illustrations

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For
Phyllis Bott
and
Lichu and Tsuchih Tu

Preface

The guiding principle in this book is to use differential forms as an aid in exploring some of the less digestible aspects of algebraic topology. Accordingly, we move primarily in the realm of smooth manifolds and use the de Rham theory as a prototype of all of cohomology. For applications to homotopy theory we also discuss by way of analogy cohomology with arbitrary coefficients.

Although we have in mind an audience with prior exposure to algebraic or differential topology, for the most part a good knowledge of linear algebra, advanced calculus, and point-set topology should suffice. Some acquaintance with manifolds, simplicial complexes, singular homology and cohomology, and homotopy groups is helpful, but not really necessary. Within the text itself we have stated with care the more advanced results that are needed, so that a mathematically mature reader who accepts these background materials on faith should be able to read the entire book with the minimal prerequisites.

There are more materials here than can be reasonably covered in a one-semester course. Certain sections may be omitted at first reading without loss of continuity. We have indicated these in the schematic diagram that follows.

This book is not intended to be foundational; rather, it is only meant to open some of the doors to the formidable edifice of modern algebraic topology. We offer it in the hope that such an informal account of the subject at a semi-introductory level fills a gap in the literature.

It would be impossible to mention all the friends, colleagues, and students whose ideas have contributed to this book. But the senior author would like on this occasion to express his deep gratitude, first of all to his primary topology teachers E. Specker, N. Steenrod, and

K. Reidemeister of thirty years ago, and secondly to H. Samelson, A. Shapiro, I. Singer, J.-P. Serre, F. Hirzebruch, A. Borel, J. Milnor, M. Atiyah, S.-s. Chern, J. Mather, P. Baum, D. Sullivan, A. Haefliger, and Graeme Segal, who, mostly in collaboration, have continued this word of mouth education to the present; the junior author is indebted to Allen Hatcher for having initiated him into algebraic topology. The reader will find their influence if not in all, then certainly in the more laudable aspects of this book. We also owe thanks to the many other people who have helped with our project: to Ron Donagi, Zbig Fiedorowicz, Dan Freed, Nancy Hingston, and Deane Yang for their reading of various portions of the manuscript and for their critical comments, to Ruby Aguirre, Lu Ann Custer, Barbara Moody, and Caroline Underwood for typing services, and to the staff of Springer-Verlag for its patience, dedication, and skill.

For the Revised Third Printing

While keeping the text essentially the same as in previous printings, we have made numerous local changes throughout. The more significant revisions concern the computation of the Euler class in Example 6.44.1 (pp. 75–76), the proof of Proposition 7.5 (p. 85), the treatment of constant and locally constant presheaves (p. 109 and p. 143), the proof of Proposition 11.2 (p. 115), a local finite hypothesis on the generalized Mayer–Vietoris sequence for compact supports (p. 139), transgressive elements (Prop. 18.13, p. 248), and the discussion of classifying spaces for vector bundles (pp. 297–300).

We would like to thank Robert Lyons, Jonathan Dorfman, Peter Law, Peter Landweber, and Michael Maltenfort, whose lists of corrections have been incorporated into the second and third printings.

RAOUL BOTT
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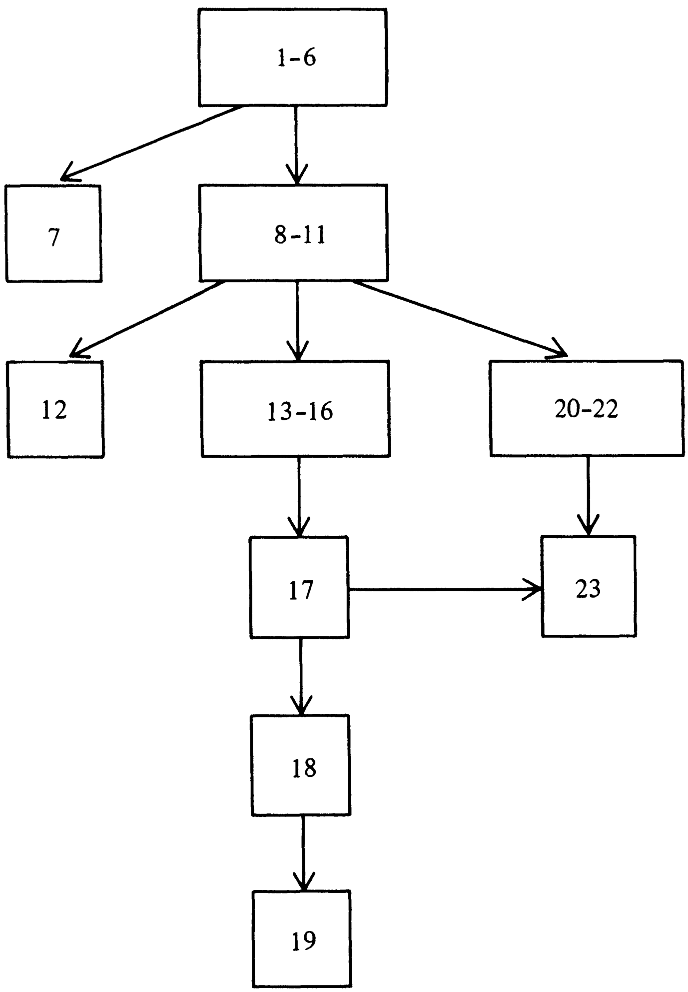
Interdependence of the Sections



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