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978-0-521-79186-1 - Irresistible Integrals: Symbolics, Analysis and Experiments in the Evaluation of Integrals

George Boros and Victor Moll

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