

Contents

<i>Preface</i>	<i>page</i>	vii
1	Classical Galois theory	1
1.1	Algebraic extensions	1
1.2	Separable extensions	4
1.3	Normal extensions	6
1.4	Galois extensions	8
2	Galois theory of Grothendieck	15
2.1	Algebras on a field	15
2.2	Extension of scalars	20
2.3	Split algebras	23
2.4	The Galois equivalence	27
3	Infinitary Galois theory	36
3.1	Finitary Galois subextensions	36
3.2	Infinitary Galois groups	39
3.3	Classical infinitary Galois theory	44
3.4	Profinite topological spaces	47
3.5	Infinitary extension of the Galois theory of Grothendieck	56
4	Categorical Galois theory of commutative rings	65
4.1	Stone duality	65
4.2	Pierce representation of a commutative ring	72
4.3	The adjoint of the ‘spectrum’ functor	80
4.4	Descent morphisms	91
4.5	Morphisms of Galois descent	98
4.6	Internal presheaves	102
4.7	The Galois theorem for rings	106

5	Categorical Galois theorem and factorization systems	116
5.1	The abstract categorical Galois theorem	117
5.2	Central extensions of groups	127
5.3	Factorization systems	144
5.4	Reflective factorization systems	149
5.5	Semi-exact reflections	156
5.6	Connected components of a space	168
5.7	Connected components of a compact Hausdorff space	170
5.8	The monotone–light factorization	177
6	Covering maps	186
6.1	Categories of abstract families	186
6.2	Some limits in $\text{Fam}(\mathcal{A})$	189
6.3	Involving extensivity	193
6.4	Local connectedness and étale maps	197
6.5	Localization and covering morphisms	201
6.6	Classification of coverings	207
6.7	The Chevalley fundamental group	212
6.8	Path and simply connected spaces	216
7	Non-galoisian Galois theory	225
7.1	Internal presheaves on an internal groupoid	225
7.2	Internal precategories and their presheaves	241
7.3	A factorization system for functors	246
7.4	Generalized descent theory	251
7.5	Generalized Galois theory	258
7.6	Classical Galois theories	261
7.7	Grothendieck toposes	266
7.8	Geometric morphisms	274
7.9	Two dimensional category theory	287
7.10	The Joyal–Tierney theorem	294
	Appendix Final remarks	304
A.1	Separable algebras	304
A.2	Back to the classical Galois theory	310
A.3	Exhibiting some links	316
A.4	A short summary of further results and developments	328
	<i>Bibliography</i>	331
	<i>Index of symbols</i>	336
	<i>General index</i>	338