

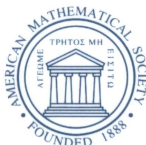
# CONTEMPORARY MATHEMATICS

242

## Geometric Aspects of Partial Differential Equations

Proceedings of a Minisymposium on  
Spectral Invariants, Heat Equation Approach  
September 18–19, 1998  
Roskilde, Denmark

Bernhelm Booss-Bavnbek  
Krzysztof Wojciechowski  
Editors



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This volume contains the proceedings of a Minisymposium on Spectral Invariants, Heat Equation Approach held at Roskilde University in Denmark on September 18–19, 1998.

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## Preface

On September 18–19, 1998, there was a Minisymposium on Spectral Invariants, Heat Equation Approach at Roskilde University in Denmark. The aim of the symposium was to discuss recent progress within the range of topics related to spectral invariants in differential geometry, quantum chromo–dynamics, and analytic number theory with emphasis on the heat equation approach. The papers in this volume are partly based on the talks given at the symposium, partly solicited after the symposium.<sup>1</sup>

The present papers provide a careful exposition of new points of view on the index, the spectral flow, the eta–invariant, the Seiberg–Witten monopoles, heat kernels, determinants, and torsion. Currently active areas of the field are introduced like spectral invariants of manifolds with boundary and of other singular manifolds, of non–compact manifolds and of infinite–dimensional manifolds. The papers reflect current interest in the use of advanced analysis methods for obtaining geometric information.

The papers are devoted to a review of developments in the subjects from an interlaced geometric and analytical point of view. Most of the papers also present new results which have not appeared previously in professional journals. With a few exceptions, these new results are presented with complete proofs.

The papers are grouped in four chapters. The focus of the *first chapter* is on the index (also construed as the chiral asymmetry of the kernel) and on small eigenvalues. The *second chapter* deals with spectral invariants characterizing the asymmetry of the spectrum of Dirac operators. The *third chapter* deals directly with the heat kernel and the derived spectral invariants. The *fourth chapter* is devoted to various generalizations of the fundamental concepts of spectral theory of elliptic operators and elliptic boundary problems.

We thank the authors for their contributions, the PDE Network and the Geometry Network of The Danish Science Research Council for their financial contribution, Roskilde University for hospitality, and the American Mathematical Society for encouragement and help in preparing this volume.

Bernhelm Booss-Bavnbek  
Krzysztof P. Wojciechowski

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<sup>1</sup>At a post–dinner discussion during the Symposium, informal recollections from the early days of index theory and pseudodifferential operators were gathered with autobiographical sketches from Bob Seeley, Misha Shubin, Peter Gilkey, Werner Müller et al. A transcript of parts of the discussion can be ordered from the editors.

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## **Geometric Aspects of Partial Differential Equations**

Bernhelm Booss-Bavnbek and Krzysztof Wojciechowski, Editors

This collection of papers by leading researchers gives a broad picture of current research directions in geometric aspects of partial differential equations. Based on lectures presented at a Minisymposium on Spectral Invariants - Heat Equation Approach, held in September 1998 at Roskilde University in Denmark, the book provides both a careful exposition of new perspectives in classical index theory and an introduction to currently active areas of the field.

Presented here are new index theorems as well as new calculations of the eta-invariant, of the spectral flow, of the Maslov index, of Seiberg-Witten monopoles, heat kernels, determinants, non-commutative residues, and of the Ray-Singer torsion. New types of boundary value problems for operators of Dirac type and generalizations to manifolds with cuspidal ends, to non-compact and to infinite-dimensional manifolds are also discussed. Throughout the book, the use of advanced analysis methods for gaining geometric insight emerges as a central theme. Aimed at graduate students and researchers, this book would be suitable as a text for an advanced graduate topics course on geometric aspects of partial differential equations and spectral invariants.

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