Graduate Texts in Mathematics

63

Béla Bollobás

Graph Theory
An Introductory Course

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Béla Bollobás
Department of Pure Mathematics
and Mathematical Statistics
University of Cambridge
16 Mill Lane
Cambridge CB2 1SB
ENGLAND

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To Gabriella

There is no permanent place in the world for ugly mathematics.

G. H. Hardy
A Mathematician's Apology

Preface

This book is intended for the young student who is interested in graph theory and wishes to study it as part of his mathematical education. Experience at Cambridge shows that none of the currently available texts meet this need. Either they are too specialized for their audience or they lack the depth and development needed to reveal the nature of the subject.

We start from the premise that graph theory is one of several courses which compete for the student's attention and should contribute to his appreciation of mathematics as a whole. Therefore, the book does not consist merely of a catalogue of results but also contains extensive descriptive passages designed to convey the flavour of the subject and to arouse the student's interest. Those theorems which are vital to the development are stated clearly, together with full and detailed proofs. The book thereby offers a leisurely introduction to graph theory which culminates in a thorough grounding in most aspects of the subject.

Each chapter contains three or four sections, exercises and bibliographical notes. Elementary exercises are marked with a $^-$ sign, while the difficult ones, marked by $^+$ signs, are often accompanied by detailed hints. In the opening sections the reader is led gently through the material: the results are rather simple and their easy proofs are presented in detail. The later sections are for those whose interest in the topic has been excited: the theorems tend to be deeper and their proofs, which may not be simple, are described more rapidly. Throughout this book the reader will discover connections with various other branches of mathematics, including optimization theory, linear algebra, group theory, projective geometry, representation theory, probability theory, analysis, knot theory and ring theory. Although most of these connections are not essential for an understanding of the book, the reader would benefit greatly from a modest acquaintance with these subjects.

The bibliographical notes are not intended to be exhaustive but rather to guide the reader to additional material.

I am grateful to Andrew Thomason for reading the manuscript carefully and making many useful suggestions. John Conway has also taught the graph theory course at Cambridge and I am particularly indebted to him for detailed advice and assistance with Chapters II and VIII. I would like to thank Springer-Verlag and especially Joyce Schanbacher for their efficiency and great skill in producing this book.

Cambridge
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Béla Bollobás

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