D. Boccaletti G. Pucacco

Theory of Orbits

Volume 2:
Perturbative and Geometrical Methods

With 81 Figures

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