

Problem Books in Mathematics

Edited by P. R. Halmos

Problem Books in Mathematics

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by *George W. Bluman*

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George W. Bluman

Problem Book for First Year Calculus

With 384 Illustrations

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To

My wife Cynthia

My sons David and Benjamin

My parents Nathan and Susan

Preface for the Student

This book focuses on the application of one-variable calculus to problems connected with physics, engineering, business, economics, biology, and chemistry. It is intended for the student who wants to “learn by doing.”

In the application of mathematics it is most important to represent and interpret visually the essentials of a given problem. Chapters I and II are concerned with problems in graphing and geometry.

Chapters III, IV, V, and VI, respectively, deal with problems pertaining to physics and engineering, business and economics, biology and chemistry, and numerical methods.

Chapters VII and VIII contain problems on the theory and techniques of calculus.

Each of the first six chapters begins with a discussion of background material necessary for doing the corresponding problems. Each chapter has a section of Solved Problems worked out in detail, often with alternative methods of solution, and a section of Supplementary Problems for which answers and occasional comments are given in Chapter IX.

In each chapter the order of problems approximates the order in which topics are encountered in most calculus courses. There are standard and difficult problems, the latter indicated by an asterisk (*). The student who masters the standard problems should have no trouble passing. The difficult problems offer a substantial challenge to the best of students.

Each chapter may be studied independently of the others. However, a student should work on Chapters I and II before proceeding to Chapters III, IV, and V.

An easy-to-use Index has each entry referenced to specific Solved and Supplementary Problems.

Preface for the Instructor

For a course in one-variable calculus this book can be used as

- (1) the textbook for a course emphasizing problem-solving;
- (2) a resource book for stimulating problems;
- (3) a supplement to your textbook.

The book contains about 1000 problems, including over 300 problems solved in great detail. There are approximately 350 diagrams. In comparison with other calculus books the emphasis is more on applied problems. Moreover, they are collected together according to the field of application. Some of the challenging problems are open-ended and use actual data (e.g., extrapolation of population data, or the speed limit for optimizing traffic flow on a bridge). The applied problems stress the concepts of calculus and require few techniques.

Many of the problems have been drawn from homework exercises and recent examination papers of various Canadian universities.

A detailed index is included.

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