

**Robert Bix**

# Conics and Cubics

A Concrete Introduction to  
Algebraic Curves

Second Edition

With 151 Illustrations

 Springer

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