

David Betounes

Differential Equations: Theory and Applications with Maple[®]



David Betounes
Mathematics Department
University of Southern Mississippi
Hattiesburg, MS 39406-5045
USA
david.betounes@usm.edu

Additional material to this book can be downloaded from <http://extras.springer.com>

Library of Congress Cataloging-in-Publication Data
Betounes, David.

Differential equations : theory and applications : with Maple / David Betounes.
p. cm.

Includes bibliographical references and index.

ISBN 978-1-4757-4973-1 ISBN 978-1-4757-4971-7 (eBook)

DOI 10.1007/978-1-4757-4971-7

1. Differential equations—Data processing. 2. Maple (Computer file). I. Title.

QA371.5D37B47 2000

515'.35'02855369—dc21

00-062030

Printed on acid-free paper.

Maple is a registered trademark of Waterloo Maple, Inc.

© 2001 Springer Science+Business Media New York
Originally published by Springer-Verlag New York, Inc. in 2001
Softcover reprint of the hardcover 1st edition 2001

This Work consists of a printed book and a CD-ROM packaged with the book, both of which are protected by federal copyright law and international treaty. The book may not be translated or copied in whole or in part without the written permission of the publisher Springer Science+Business Media, LLC, except for brief excerpts in connection with reviews or scholarly analysis. For copyright information regarding the CD-ROM, please consult the printed information packaged with the CD-ROM in the back of this publication, and which is also stored as a "readme" file on the CD-ROM. Use of the printed version of this Work in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known, or hereafter developed, other than those uses expressly granted in the CD-ROM copyright notice and disclaimer information, is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone. Where those designations appear in the book and Springer-Verlag was aware of a trademark claim, the designations follow the capitalization style used by the manufacturer.

Production managed by Steven Pisano; manufacturing supervised by Jerome Basma.
Photocomposed pages prepared from the author's LaTeX files.

9 8 7 6 5 4 3 2 1

SPIN 10780636

Contents

Preface	v
1 Introduction	1
1.1 Examples of Dynamical Systems	1
1.2 Vector Fields and Dynamical Systems	14
1.3 Nonautonomous Systems	23
1.4 Fixed Points	26
1.5 Reduction to 1st-Order, Autonomous	27
1.6 Summary	32
2 Techniques, Concepts, and Examples	33
2.1 Euler's Numerical Method	34
2.1.1 The Geometric View	34
2.1.2 The Analytical View	36
2.2 Gradient Vector Fields	39
2.3 Fixed Points and Stability	45
2.4 Limit Cycles	51
2.5 The Two-Body Problem	55
2.5.1 Jacobi Coordinates	57
2.5.2 The Central Force Problem	59
2.6 Summary	72
3 Existence and Uniqueness: The Flow Map	75
3.1 Picard Iteration	78
3.2 Existence and Uniqueness Theorems	82
3.3 <i>Maximum Interval of Existence</i>	92
3.4 The Flow Generated by a Time-Dependent Vector Field	95

3.5	The Flow for Autonomous Systems	104
3.6	Summary	112
4	One-Dimensional Systems	115
4.1	Autonomous, One-Dimensional Systems	116
4.1.1	Construction of the Flow for 1-D, Autonomous Systems	123
4.2	Separable Differential Equations	128
4.3	Integrable Differential Equations	135
4.4	Homogeneous Differential Equations	147
4.5	Linear and Bernoulli Differential Equations	151
4.6	Summary	155
5	Linear Systems	157
5.1	Existence and Uniqueness for Linear Systems	162
5.2	The Fundamental Matrix and the Flow	165
5.3	Homogeneous, Constant Coefficient Systems	174
5.4	The Geometry of the Integral Curves	180
5.4.1	Real Eigenvalues	182
5.4.2	Complex Eigenvalues	192
5.5	Canonical Systems	211
5.5.1	Diagonalizable Matrices	214
5.5.2	Complex Diagonalizable Matrices	217
5.5.3	The Nondiagonalizable Case: Jordan Forms	219
5.6	Summary	227
6	Linearization and Transformation	231
6.1	Linearization	231
6.2	Transforming Systems of DEs	247
6.2.1	The Spherical Coordinate Transformation	253
6.2.2	Some Results on Differentiable Equivalence	257
6.3	The Linearization and Flow Box Theorems	266
7	Stability Theory	275
7.1	Stability of Fixed Points	276
7.2	Linear Stability of Fixed Points	279
7.2.1	Computation of the Matrix Exponential for Jordan Forms	280
7.3	Nonlinear Stability	290
7.4	Liapunov Functions	292

7.5	Stability of Periodic Solutions	303
8	Integrable Systems	323
8.1	First Integrals (Constants of the Motion)	324
8.2	Integrable Systems in the Plane	329
8.3	Integrable Systems in 3-D	334
8.4	Integrable Systems in Higher Dimensions	348
9	Newtonian Mechanics	361
9.1	The N -Body Problem	362
9.1.1	Fixed Points	365
9.1.2	Initial Conditions	366
9.1.3	Conservation Laws	366
9.1.4	Stability of Conservative Systems	374
9.2	Euler's Method and the N -body Problem	384
9.2.1	Discrete Conservation Laws	392
9.3	The Central Force Problem Revisited	401
9.3.1	Effective Potentials	404
9.3.2	Qualitative Analysis	405
9.3.3	Linearization and Stability	409
9.3.4	Circular Orbits	410
9.3.5	Analytical Solution	412
9.4	Rigid-Body Motions	424
9.4.1	The Rigid-Body Differential Equations	432
9.4.2	Kinetic Energy and Moments of Inertia	438
9.4.3	The Degenerate Case	446
9.4.4	Euler's Equation	447
9.4.5	The General Solution of Euler's Equation	451
10	Motion on a Submanifold	463
10.1	Motion on a Stationary Submanifold	464
10.1.1	Motion Constrained to a Curve	471
10.1.2	Motion Constrained to a Surface	476
10.2	Geometry of Submanifolds	484
10.3	Conservation of Energy	493
10.4	Fixed Points and Stability	495
10.5	Motion on a Given Curve	502
10.6	Motion on a Given Surface	513
10.6.1	Surfaces of Revolution	520
10.6.2	Visualization of Motion on a Given Surface	526

10.7 Motion Constrained to a Moving Submanifold	531
11 Hamiltonian Systems	541
11.1 1-Dimensional Hamiltonian Systems	544
11.1.1 Conservation of Energy	547
11.2 Conservation Laws and Poisson Brackets	551
11.3 Lie Brackets and Arnold's Theorem	565
11.3.1 Arnold's Theorem	567
11.4 Liouville's Theorem	582
A Elementary Analysis	589
A.1 Multivariable Calculus	589
A.2 The Chain Rule	595
A.3 The Inverse and Implicit Function Theorems	596
A.4 Taylor's Theorem and The Hessian	602
A.5 The Change of Variables Formula	606
B Lipschitz Maps and Linearization	607
B.1 Norms	608
B.2 Lipschitz Functions	609
B.3 The Contraction Mapping Principle	613
B.4 The Linearization Theorem	619
C Linear Algebra	633
C.1 Vector Spaces and Direct Sums	633
C.2 Bilinear Forms	636
C.3 Inner Product Spaces	638
C.4 The Principal Axes Theorem	642
C.5 Generalized Eigenspaces	645
C.6 Matrix Analysis	656
C.6.1 Power Series with Matrix Coefficients	662
D CD-ROM Contents	665
Bibliography	669
Index	675