Applied Mathematical Sciences
Volume 151

Springer-Verlag
Berlin Heidelberg GmbH

Applied Mathematical Sciences

1. *John:* Partial Differential Equations, 4th ed.
2. *Sirovich:* Techniques of Asymptotic Analysis.
3. *Hale:* Theory of Functional Differential Equations, 2nd ed.
4. *Percus:* Combinatorial Methods.
5. *von Mises/Friedrichs:* Fluid Dynamics.
6. *Freiberger/Grenander:* A Short Course in Computational Probability and Statistics.
7. *Pipkin:* Lectures on Viscoelasticity Theory.
8. *Giacaglia:* Perturbation Methods in Non-linear Systems.
9. *Friedrichs:* Spectral Theory of Operators in Hilbert Space.
10. *Stroud:* Numerical Quadrature and Solution of Ordinary Differential Equations.
11. *Wolovich:* Linear Multivariable Systems.
12. *Berkovitz:* Optimal Control Theory.
13. *Bluman/Cole:* Similarity Methods for Differential Equations.
14. *Yoshizawa:* Stability Theory and the Existence of Periodic Solution and Almost Periodic Solutions.
15. *Braun:* Differential Equations and Their Applications, 3rd ed.
16. *Lefschetz:* Applications of Algebraic Topology.
17. *Collatz/Wetterling:* Optimization Problems.
18. *Grenander:* Pattern Synthesis: Lectures in Pattern Theory, Vol. I.
19. *Marsden/McCracken:* Hopf Bifurcation and Its Applications.
20. *Driver:* Ordinary and Delay Differential Equations.
21. *Courant/Friedrichs:* Supersonic Flow and Shock Waves.
22. *Rouche/Habets/Laloy:* Stability Theory by Liapunov's Direct Method.
23. *Lamperti:* Stochastic Processes: A Survey of the Mathematical Theory.
24. *Grenander:* Pattern Analysis: Lectures in Pattern Theory, Vol. II.
25. *Davies:* Integral Transforms and Their Applications, 2nd ed.
26. *Kushner/Clark:* Stochastic Approximation Methods for Constrained and Unconstrained Systems.
27. *de Boor:* A Practical Guide to Splines, Rev. ed.
28. *Keilson:* Markov Chain Models—Rarity and Exponentiality.
29. *de Veubeke:* A Course in Elasticity.
30. *Sniatycki:* Geometric Quantization and Quantum Mechanics.
31. *Reid:* Sturmian Theory for Ordinary Differential Equations.
32. *Meis/Markowitz:* Numerical Solution of Partial Differential Equations.
33. *Grenander:* Regular Structures: Lectures in Pattern Theory, Vol. III.
34. *Kevorkian/Cole:* Perturbation Methods in Applied Mathematics.
35. *Carr:* Applications of Centre Manifold Theory.
36. *Bengtsson/Ghil/Källén:* Dynamic Meteorology: Data Assimilation Methods.
37. *Saperstone:* Semidynamical Systems in Infinite Dimensional Spaces.
38. *Lichtenberg/Lieberman:* Regular and Chaotic Dynamics, 2nd ed.
39. *Piccini/Stampacchia/Vidossich:* Ordinary Differential Equations in $\mathbf{R}^n$.
40. *Naylor/Sell:* Linear Operator Theory in Engineering and Science.
41. *Sparrow:* The Lorenz Equations: Bifurcations, Chaos, and Strange Attractors.
42. *Guckenheimer/Holmes:* Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields.
43. *Ockendon/Taylor:* Inviscid Fluid Flows.
44. *Pazy:* Semigroups of Linear Operators and Applications to Partial Differential Equations.
45. *Glashoff/Gustafson:* Linear Operations and Approximation: An Introduction to the Theoretical Analysis and Numerical Treatment of Semi-Infinite Programs.
46. *Wilcox:* Scattering Theory for Diffraction Gratings.
47. *HaleMagalhaes/Oliva:* Dynamics in Infinite Dimensions, 2nd ed.
48. *Murray:* Asymptotic Analysis.
49. *Ladyzhenskaya:* The Boundary-Value Problems of Mathematical Physics.
50. *Wilcox:* Sound Propagation in Stratified Fluids.
51. *Golubitsky/Schaeffer:* Bifurcation and Groups in Bifurcation Theory, Vol. I.
52. *Chipot:* Variational Inequalities and Flow in Porous Media.
53. *Majda:* Compressible Fluid Flow and System of Conservation Laws in Several Space Variables.
54. *Wasow:* Linear Turning Point Theory.
55. *Yosida:* Operational Calculus: A Theory of Hyperfunctions.
56. *Chang/Howes:* Nonlinear Singular Perturbation Phenomena: Theory and Applications.
57. *Reinhardt:* Analysis of Approximation Methods for Differential and Integral Equations.
58. *Dwoyer/Hussaini/Voigt (eds):* Theoretical Approaches to Turbulence.
59. *Sanders/Verhulst:* Averaging Methods in Nonlinear Dynamical Systems.

(continued following index)

Alain Bensoussan Jens Frehse

Regularity Results for Nonlinear Elliptic Systems and Applications

Springer

Alain Bensoussan
CNES
2 Place Maurice Quentin
75039 Paris, France
alain.bensoussan@cnes.fr

Jens Frehse
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstrasse 10
53115 Bonn, Germany

Mathematics Subject Classification (2000):
35-XX, 49-XX, 74-XX, 91A15, 76-XX

Cataloging-in-Publication data applied for

Die Deutsche Bibliothek – CIP-Einheitsaufnahme

Bensoussan, Alain: Regularity results for nonlinear elliptic systems and applications /
Alain Bensoussan; Jens Frehse.
(Applied mathematical sciences; Vol. 151)
ISBN 978-3-642-08726-4 ISBN 978-3-662-12905-0 (eBook)
DOI 10.1007/978-3-662-12905-0
0101 deutsche buecherei

ISBN 978-3-642-08726-4

http://www.springer.de

Originally published by Springer-Verlag Berlin Heidelberg New York in 2002
Softcover reprint of the hardcover 1st edition 2002

TEX-editing by H.-J. Wolf and Educational Media Centre, New Delhi, India
SPIN 10701624 41/3142 - 5 4 3 2 1 0 – Printed on acid-free paper

Preface

Nonlinear elliptic equations and systems are a classical field of analysis, with many applications in differential geometry, continuum mechanics, and probability theory; an important future branch will be their applications to microelectronics.

The most important analytical tools in the field of nonlinear partial differential equations and systems up to, say, 1955 are presented in the books of C. B. Morrey [79] and O.A. Ladyzhenskaya, N.N. Ural'tseva [66]. The bulk of the development for general nonlinear elliptic systems is presented in M. Giaquinta, E. Giusti [41], D. Gilbarg, N.S. Trudinger [46], later in M. Giaquinta [40]. Concerning applications to differential geometry, we mention the books of M. Giaquinta, S. Hildebrandt [42].

The purpose of this book is to present some of the developments that are not covered in the above books and are promising fields for applications and research.

The book is to a large extent self-contained, with the restriction that the linear theory—Schauder estimates and Campanato theory—is not presented. The reader is expected to be familiar with functional-analytic tools, like the theory of monotone operators. References are given in the text to any techniques that are used. The first two chapters contain general methods and auxiliary lemmas. The expert might like our approach to the theorem of De Giorgi–Nash concerning C^{α}-regularity of solutions to nonlinear scalar equations via the hole-filling method, and our proof of Harnack's inequality without using the John–Nirenberg theorem on functions with bounded mean oscillation.

Chapters 4 and 5 deal with diagonal elliptic systems, which have important applications to differential geometry; however, in order to be complementary to the books of Giaquinta–Hildebrandt, we present only the applications to stochastic problems, where the researcher finds challenging open problems with a broad range of degree of difficulty. In fact, the treatment here is more complete than what is available in the literature.

Chapter 6 deals with Helein's proof of the regularity of harmonic mappings on two-dimensional manifolds. We avoid a more extensive study of harmonic mappings, for which we refer to the books of J. Jost [60], M. Giaquinta, S. Hildebrandt [42] (see also J. Eells, J.H. Sampson [22]).

Chapter 7 presents the standard Van Roosbroek equations in semi-conductor theory and a special model that is related to the avalanche effect. We admit that this choice represents a limited sample compared with the range of interesting new open problems waiting to be solved, but in the interest of brevity we have cut the exposition short. In chapter 8 we present recent results for the regularity problem of the Navier–Stokes equation. Clearly, this chapter is not an introduction to mathematical fluid dynamics, for which the reader should refer to the standard book of O.A. Ladyzhenskaya, V.A. Solonnikov, N.N. Ural'tseva [67] or, recently, P.L. Lions [72] and G.P. Galdi [36]. We have included this chapter in the book because of surprising similarities of the analytical tools to those in the chapter on diagonal systems. In Chapter 9 we collect results concerning strongly coupled elliptic systems, in particular the theory of A. Koshelev [63] concerning sufficient conditions for regularity involving eigenvalues. Chapter 10 presents elements of a dual theory of elliptic systems, the motivation coming from simple models in elasto plasticity. It seems that many techniques in elliptic analysis have a dual analogue. For example, we present a dual proof and formulation of the almost everywhere regularity of solutions of elliptic systems. Chapter 11 contains a short approach to plasticity theory; for the physical background we refer to the books of G. Duvaut, J.L. Lions [15], R. Temam [101] and P. Le Tallec [69]. We believe that the approach via the Norton–Hoff approximation is a recommendable introduction for newcomers who have knowledge of Sobolev spaces. We would like to emphasize that much of the progress concerning the time-dependent Prandtl–Reuss law and regularity properties of its solution has been made by using the dual theory of elliptic equations. This is why it is presented here, although it is a "time"-dependent model, which is in principle outside the scope of this book.

We would like to thank Zamin Iqbal, who carefully read the draft of the book and improved the English to a great extent, and also Josef Malek, who read various parts.

A warm thank you to Chantal Delabarre, who improved the limited LaTeX of the authors, and to Springer-Verlag for publishing this book.

Alain Bensoussan
Jens Frehse

Contents