Series on Advances in Mathematics for Applied Sciences – Vol. 19

A CONCISE GUIDE TO SEMIGROUPS AND EVOLUTION EQUATIONS

Aldo Belleni-Morante
Dipartimento di Ingegneria Civile
Università di Firenze

World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 73 Lynton Mead, Totteridge, London N20 8DH

A CONCISE GUIDE TO SEMIGROUPS AND EVOLUTION EQUATIONS

Copyright © 1994 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 27 Congress Street, Salem, MA 01970, USA.

ISBN 981-02-1294-1

Printed in Singapore by Utopia Press.

CONTENTS

PREFACE VII

1. TWO TYPICAL EVOLUTION PROBLEMS 1

1.1. A linear problem 1

Note 1.1 Strong continuity 6

Note 1.2. Generalized derivatives 7

1.2. A semilinear problem 8

Note 1.3. Subspaces of a Banach space 12

2. SEMIGROUPS GENERATED BY BOUNDED OPERATORS 14

2.1. Time independent bounded operators 14

Note 2.1. Convergence of a sequence of operators 21

Note 2.2. Some technicalities 23

Note 2.3. Two different strong derivatives 24

Note 2.4. Gronwall's lemma 25

2.2. Time dependent bounded operators 26

3. SEMIGROUPS GENERATED BY $A \in G(1,0;X)$ 35

3.1. Introduction 35

3.2. The evolution problem (41) 43

Note 3.1. Dense subset 50

Note 3.2. Closed operators 51

Note 3.3. $D(A^2)$ as a dense subset of X 52

Note 3.4. The vector space C_0^∞ 53

Note 3.5. Hilbert spaces 53

4. THE CASES $A \in G(M,\beta;X)$ AND $A \in G'(1,0;X)$ 55

4.1. The case $A \in G(M,\beta;X)$ 55

4.2. The case $A \in G(1,\beta;X)$ 57

Note 4.1. Hölder inequality 59

4.3. The abstract evolution problem with $A \in G(M,\beta;X)$ 60

4.4. Positive solutions 61

Note 4.2. Positivity of the strong Riemann integral 65

4.5. The case $A \in G'(1,0;X)$ 65

5. BOUNDED PERTURBATIONS 69

5.1. Introduction 69

5.2. Bounded perturbations 70

Note 5.1. Norm of the product of two bounded operators 73

Note 5.2. The operator $(\mu I - B_0)^{-1}$ with $B_0 \in \mathcal{B}(X)$ 73

5.3. The integral version of system (78) 74

Note 5.3. Bounded operators defined on the same dense set 78

6. HOLOMORPHIC SEMIGROUPS 79

6.1. Introduction 79

6.2. The semigroup generated by $A \in \mathcal{H}(\alpha,M,0;X)$ 81

Note 6.1. Line integrals 86
6.3. Further properties of $T(t)$ 87
6.4. The homogeneous and the nonhomogeneous evolution problems 93

7. A LINEAR EVOLUTION PROBLEM 96
7.1. Introduction 96
7.2. The operators T, B and K 97
7.3. Solution of the abstract problem (105) 100

8. SEMILINEAR EVOLUTION PROBLEMS 106
8.1. Introduction 106
Note 8.1. Lipschitz operators 110
8.2. The nonlinear integral equation (120) 111
Note 8.2. Convex sets 115
Note 8.3. Frechét derivative 116
Note 8.4. Contraction mapping theorem 118
8.3. Strict and mild solutions of system (115) 118
Note 8.5. Some technicalities 127
Note 8.6. Bounded convergence theorem 129
Note 8.7. Some properties of $A_1 \in G(M,\beta;X)$ 129
8.4. Global in time solutions of (115) 130
8.5. A semilinear problem from populationn theory: solution local in time 131
8.6. A semilinear problem from population theory: positivity properties 136
8.7. A semilinear problem from population theory: solution global in time, part I 140
8.8. A semilinear problem from population theory: solution global in time, part II 142

8.9. Some remarks on the nonhomogeneous case 146

APPENDIX A: Lebesgue measure and Lebesgue integral 149
APPENDIX B: Banach spaces 153
REFERENCES 158
SUBJECT INDEX 161