

Contents

<i>Preface</i>	<i>page</i> ix
<i>Acknowledgements</i>	xi
Introduction	1
1 Basic features of smooth worlds	16
2 Basic differential calculus	24
2.1 The derivative of a function	24
2.2 Stationary points of functions	27
2.3 Areas under curves and the Constancy Principle	28
2.4 The special functions	30
3 First applications of the differential calculus	35
3.1 Areas and volumes	35
3.2 Volumes of revolution	40
3.3 Arc length; surfaces of revolution; curvature	43
4 Applications to physics	49
4.1 Moments of inertia	49
4.2 Centres of mass	54
4.3 Pappus' theorems	55
4.4 Centres of pressure	58
4.5 Stretching a spring	60
4.6 Flexure of beams	60
4.7 The catenary, the loaded chain and the bollard-rope	63
4.8 The Kepler–Newton areal law of motion under a central force	67

viii	<i>Contents</i>	
5	Multivariable calculus and applications	69
5.1	Partial derivatives	69
5.2	Stationary values of functions	72
5.3	Theory of surfaces. Spacetime metrics	75
5.4	The heat equation	81
5.5	The basic equations of hydrodynamics	82
5.6	The wave equation	84
5.7	The Cauchy–Riemann equations for complex functions	86
6	The definite integral. Higher-order infinitesimals	89
6.1	The definite integral	89
6.2	Higher-order infinitesimals and Taylor’s theorem	92
6.3	The three natural microneighbourhoods of zero	95
7	Synthetic differential geometry	96
7.1	Tangent vectors and tangent spaces	96
7.2	Vector fields	98
7.3	Differentials and directional derivatives	98
8	Smooth infinitesimal analysis as an axiomatic system	102
8.1	Natural numbers in smooth worlds	108
8.2	Nonstandard analysis	110
	<i>Appendix. Models for smooth infinitesimal analysis</i>	113
	<i>Note on sources and further reading</i>	119
	<i>References</i>	121
	<i>Index</i>	123