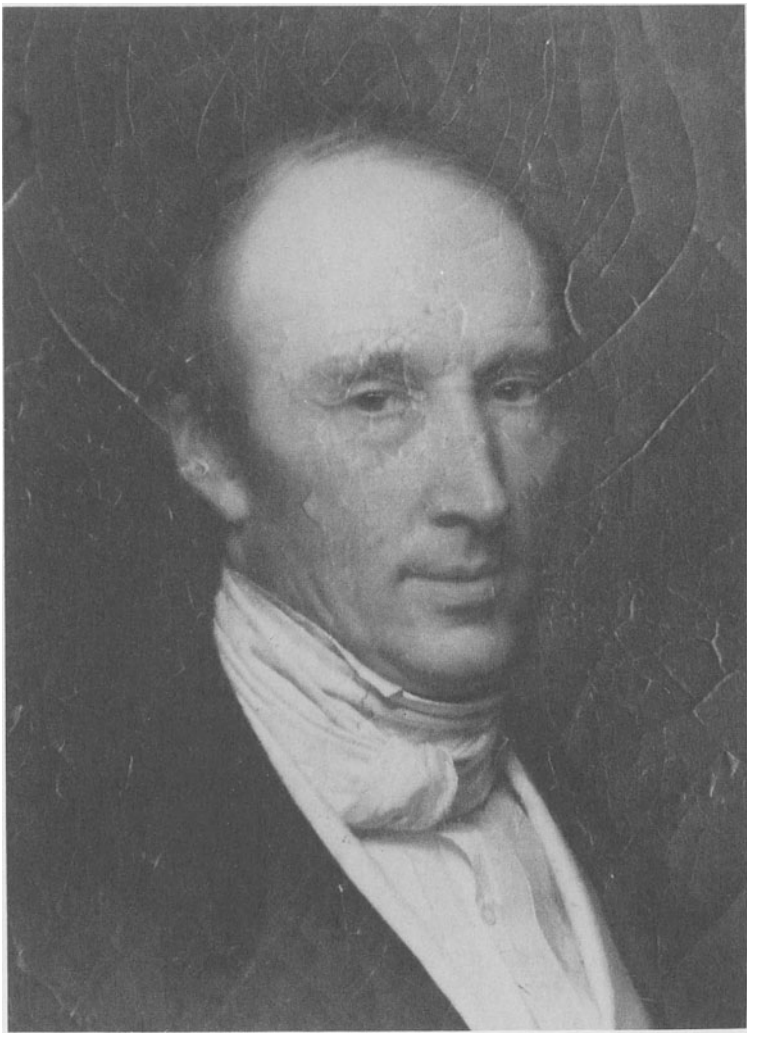
Augustin-Louis Cauchy



Painting of Augustin-Louis Cauchy by J. Roller ($\simeq$ 1840).

Bruno Belhoste

Augustin-Louis Cauchy

A Biography

Translated by Frank Ragland

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Foreword

To write a biography about a leading scientific figure is admittedly an ambitious undertaking for a historian, since, in addition to the usual difficulties presented by biographical studies, one must now contend with the strange intricacies of scientific research and thought. This difficulty is compounded considerably in at least two ways for one who would write a biography of Augustin-Louis Cauchy (1789–1857). First of all, the vast output of Cauchy's creative genius can be overwhelming on its own terms—even for mathematicians. Second, mathematical notation, concepts, and terminology were far from logical or uniform in Cauchy's day, and thus the historian faces a double challenge in pursuing the development of scientific ideas and the relationships between them. An equally difficult (but no less important) task facing a biographer of Cauchy is that of delineating the curious interplay between the man, his times, and his scientific endeavors. I think Professor Belhoste has succeeded admirably in meeting all of these challenges and has thus written a vivid biography that is both readable and informative.

Professor Belhoste's subject stands out as one of the most brilliant, versatile, and prolific figures in the annals of science. Cauchy was a man who, though a creator of our times, was nevertheless very much a creation of his own age, a particularly dynamic period in Western history that has come to be known as the Age of Revolutions (1789–1848). Although the revolutions of that era may have been primarily political upheavals, they were also scientific in the important sense that it is during this period that science and scientific scholarship passed from the control of a few enlightened despots and aristocrats into the hands of the state and its appendages, the universities and other forerunners of today's research institutions. It can, of course, be debated whether this professionalization of science was really revolutionary or merely evolutionary in the sense of a clearly discernable shift in how, where, by whom, and under whose auspices systematic scientific investigations should be carried out. However, what Professor Belhoste's work makes admirably clear is that by the time Cauchy became established in the world of science, these issues had already been fairly well decided.

Nearly two hundred years have now passed since the young Cauchy set about his task of clarifying mathematics, extending it, and applying it (whenever possible) and placing it on a firm theoretical footing. Even as he reacted to his society's sudden shifts and turns—political, religious, and intellectual—in ways which may seem to us today as hasty and ill-considered, he doggedly pursued this youthful vision. In doing so, he made many fundamental contributions not only to mathematics but to physics and astronomy as well. The true measure of Cauchy's success in the grand undertaking of his youth must necessarily be sought in the standards and methods that he bequeathed to modern science. If he was not wholly successful in the task he originally set for himself, his lack of success would seem to have less to do with any shortcomings or defects on his part than with the stupendous vastness and subtlety of mathematics and its related disciplines. And this, too, is yet another level on which Professor Belhoste's work achieves practical importance; for here we are afforded a detailed, rather personalized picture of how a first-rate mathematician worked at his discipline—his strivings, his inspirations, his triumphs, his failures, and above all, his conflicts and his errors. In this respect, then, this study should be of signal interest to young students of the mathematical sciences, since, all too often, they only catch a glimpse of Cauchy as a supremely confident and creative genious and not as a human being endowed with his share of the errors, weaknesses, and shortcomings that are such an important part of human nature.

In translating this work, I have tried to strike a balance between the flavor of the original French study and requirements of readability. To this end, in the body of the text I have given the French titles of Cauchy's works (and for those of his contemporaries also). Similarly, the original French titles are used exclusively in the bibliographical notes, and this applies also to the works of other authors mentioned in the course of the text. In this way, any reader desiring to consult the originals will have little difficulty in doing so. Similarly, I have taken care to maintain the French system of citation used by the author.

Finally, I would like to thank Mss. Connie Burgess and Joan Passalacqua for their patience and steadfast devotion in typing the manuscript.

Frank Ragland
New York
Winter, 1989

Preface

Cauchy was the true heir of the great mathematical tradition of the 18th century, the heir of Euler, Legendre, Lagrange, and Laplace; and yet he was both a leading standard-bearer and an active creator of an essentially new approach to mathematics. Indeed, Cauchy and Karl Friedrich Gauss (1777–1855) may rightly be called the first truly modern mathematicians. The sheer bulk of his scientific productivity is immense. The *Oeuvres Complètes*, a publication which covers almost an entire century, from 1882 to 1975, fills more than 27 large volumes and contains, in addition to five complete textbooks, nearly 800 research articles and treatises. Cauchy's work does contain an element of redundancy. But, on balance, such an enormous scientific creativity is nothing less than staggering, for it presents research on all the then-known areas of mathematics: arithmetic, algebra, geometry, statistics, mechanics, real and complex analysis, and mathematical physics. Nevertheless, in spite of its vastness and rich multifaceted character, Cauchy's scientific works possess a definite unifying theme, a secret wholeness. This, at least, is the essential point of departure of the present study.

There are several ways in which a study of this type might have been developed. A historian by training and by temperament, I have chosen the most natural method: the biography. To be sure, there already exists a biography of Cauchy. In 1868, eleven years after Cauchy's death, C.A. Valson, a mathematician of Catholic persuasion who was preparing the publication of the *Oeuvres Complètes*, published a biography.[1] Published under the auspices of the Cauchy family, this two-volume study is not without interest today. This is mainly so because Valson had at his disposal certain of Cauchy's personal papers—documents which have now been completely destroyed—as well as the family's archives and the actual testimony of many of Cauchy's associates. In spite of this, however, Valson's study fails to meet the standards of rigor and scholarship demanded by modern historical studies. Replete with irrelevant

[1] *La Vie et les Travaux du Baron Cauchy* (2 volumes, Paris, 1868).

moral judgements, and frequently succinct on many aspects of the great mathematician's life and works, Valson's work would seem to be more hagiography than history.

In the present work, I have earnestly sought to present a portrait of a mathematician and the age in which he lived. In this way, I hope to underscore, to highlight, as it were, the essential coherence of an active, productive career in the sciences which stretched over nearly half a century. Certainly, Augustin-Louis Cauchy was a far cry from the romantic heros of that era. His long and active life lacks the ephemeral sparkle and brilliance of that of his contemporary Evariste Galois. A devout Catholic and close associate of the Jesuits as well as a strict royalist, Cauchy quickly became a recognized mathematician who, at a very early age, took a place in the leading scientific institutions of his day. On the other hand, however, he was a proud man, a man of passionately held convictions who, whenever occasions arose for him to defend or to explain those things that he regarded as "the truth," consistently refused to allow himself to be swayed by considerations of personal convenience or self-interest. Thus, for example, in the political sphere his adherence to the Bourbon cause, was, for better or worse, absolute and unyielding. Caught up in the political purges of the 1830s, though he himself had earlier profited from similar procedures in 1816 after the fall of Bonaparte, Cauchy chose exile over perjury. In fact, from 1830 until his death in 1857, he steadfastly refused to swear loyalty oaths to any of the regimes that governed France. In a similar way, Cauchy's belief in Catholicism was uncompromising and apparently untarnished by any doubts whatsoever. Indeed, throughout his life, Cauchy practiced his faith with all the zeal of a new convert and with all the feverish passion of a missionary.

Cauchy's view of mathematics sprang from the same deeply felt need for absolutes, for certainty. If, in politics, this need manifested itself in uncompromising adherence to the Bourbon cause and in religion to strict Roman Catholic orthodoxy, then in mathematics its expression was reflected in a demand for rigor and irrefragable proof. To Cauchy, the real work of a scholar, of a scientist, must necessarily be a quest for truth. "Truth," he wrote in 1842, "is a priceless treasure which, whenever we manage to acquire it, cannot bring us remorse and sorrow; it cannot disquiet and distress our soul. The mere thought of its heavenly attributes, of its divine beauty suffices to replenish us for all the sacrifices we may have made in discovering it. Indeed, the joy of heaven itself is but the full and complete possession of immortal truth." Feeling uncomfortable in the age in which he lived and often misunderstood by his contemporaries, Cauchy found a refuge in mathematics. Here, in a world far removed from the everyday one, his creative genius could thrive, expand, and reach its full measure of expression.

Still, in a sense, his view of mathematics represents a certain expression of the concerns of the era in which he lived. Thus it was that Cauchy early rejected the optimistic rationalism of the Age of the Enlightment in no uncertain terms even though Laplace, his mentor and protector, was

ever faithful to the credo of the philosophes. As it specifically concerns mathematics, it should be noted that analysis had experienced tremendous growth all during the 18th century. Unfortunately, this growth had come about at the expense of mathematical rigor; and Cauchy, like the more able of his associates and contemporaries, set about strengthening the theoretical foundations of mathematics and restructuring the entire edifice. His position as a professor of mathematics at the École Polytechnique provided him with an excellent forum for working towards these ends. Certainly, his demand for rigor in mathematics proved to be exceedingly fruitful, since it opened up wholly new rich fields of mathematical research.

But Cauchy's creative genius found broad expression not only in his work on the foundations of real and complex analysis areas to which his name is inextricably linked, but also in many other fields. Specifically, in this connection, we should mention his major contributions to the development of mathematical physics and to theoretical mechanics, two fields which experienced rapid growth during the 19th century. Along these lines we mention, among others, his two theories of elasticity and his investigations on the theory of light, research which required that he develop whole new mathematical techniques such as Fourier transforms, diagonalization of matrices, and the calculus of residues.

It should be observed that as to his mathematical talents, Cauchy—like Euler and Gauss—was a universalist in the fullest sense of the term. His few works on the theory of numbers, algebra, and geometry make us deeply regret that he did not devote more effort to these areas. It can hardly be doubted that had he done so, he would have obtained results of signal importance. Cauchy's creativity, however, bore the stamp of his training as an engineer. Accordingly, Cauchy, like other great mathematicians of the French School, always gave priority to questions about applied mathematics which were a great source of inspiration for him.

Today, two centuries after Cauchy's birth, his work has been completely integrated into the austerely beautiful and elegant structure of mathematics, a structure which is constantly changing. And thus it is that Cauchy's theorems and theories have been reformulated in newer, more modern terms and the memory of the scholar himself has been progressively weakened. Only his name attached to a few outstanding results remains to give testimony to future generations of the importance of his works. It is the fate of all mathematicians to see their individual contributions become quickly absorbed into the great common structure. And while this is hardly the place to dwell on this simple fact, we should nevertheless pause and consider another simple fact: mathematics—regardless of how impersonal it may be in its formal beauty and power, is not something handed down complete and perfect from heaven. Rather it is the cumulative result of the work (and, quite frequently, of the sufferings also) of many individual human beings. If, somehow, this biography of Cauchy should help in establishing this belief, then I will have attained my goal.

This book was the outcome of several years of work, but it could not have been completed without the help of a number of persons. First of all, I am deeply grateful to Professor René Taton, who directed my initial research with great care and kindness. I must also express my gratitude to those who were kind enough to provide me with often rare documents and information. In this respect, I am particularly indebted to Professors Dugac, Grattan-Guinness, Ross, Russo, and Yuschkevich. I also recall with special gratitude and fondness the very fruitful discussion that I had with my good friends Amy Dahan-Dalmedico and Jesper Lützen. Nor can I forget the warm reception I was always accorded at the archives and at the libraries. Special mention must be given to the Secretariat of the Academy of Sciences where M. Berthon and Mme Pouret kindly placed at my disposal their profound knowledge of the archives. Similarly, I was greatly assisted by the cogent advice of Mlle Billoux at the Central Library of the École Polytechnique. I also want to express my gratitude to Professor Frank Ragland of the City University of New York who translated this work into English with great skill and care. Finally, I should like to thank Jeremy Gray and John Greenberg who carefully read the manuscript. Their comments enabled me to make a number of changes in the final text.

Paris, France
Autumn, 1989

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