

Lecture Notes in Mathematics

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Topics in Set Theory

Lebesgue Measurability, Large Cardinals,
Forcing Axioms, Rho-functions

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During the Fall 1987 Professor Stevo Todorcevic gave a series of lectures for the Ulam Seminar at the University of Colorado at Boulder. This is a reworked version of my handwritten notes taken during the Seminar. I have added a few supplementary results found in the well-known texts listed in the references and added a few details in some of the proofs, but the basic sequence of results remains unchanged. Since the set of notes lacked any historical discussion, only a partial list of references has been supplied at the end of each chapter.

I would like to thank Professor Stevo Todorcevic for a large amount of time spent after the Seminar in explaining the proofs to me and for the permission of including here a number of his unpublished notes. This work also greatly depended on Liz Stimmel who, with utmost patience and care, typed the manuscript.

M. Bekkali
Boulder; January 1991

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This chapter deals with various examples of nonmeasurable sets of reals. Also a definable (in fact Σ_3^1) nonmeasurable set of reals is given assuming that ω_1 is not inaccessible in L . In other words if all projective sets of reals are measurable, then ω_1 is an inaccessible cardinal in L .

Chapter 2: Measurability in $L[\mathbb{R}]$	15–22
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The assumption ω_1 is not inaccessible in L , in the previous chapter, is a too strong restriction on the universe of sets. So, it cannot be considered as an answer to the old problem: Does there exist a definable nonmeasurable set of reals? By contrast with measurable cardinals, we'll see that supercompact cardinals settle this problem completely. The main ingredient in proving this result is the notion of a saturated ideal over ω_1 .

Chapter 3: Forcing Axioms	23–60
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It is proved that MA is a Ramsey-type statement. PFA and SPFA are introduced and it is proved that PFA implies $2^{\aleph_0} = \aleph_2$. Applications are given to Boolean algebras, real line and Banach algebras.

Chapter 4: The Method of Minimal Walks	61–104
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The technique of minimal walks, which is used to prove results in ZFC in some cases where CH or some other enumeration principle is used, is studied carefully. Applications to squares, Aronszajn trees and partition relations are given. Some of these later are stepped-up to higher cardinalities.

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Most of the results of the Appendix have not been mentioned during the lectures. They are included here because I feel that they might serve as a background for better understanding of Chapter 2.

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