Die Grundlehren der mathematischen Wissenschaften

in Einzeldarstellungen
mit besonderer Berücksichtigung
der Anwendungsgebiete

Band 201

Herausgegeben von

S. S. Chern J. L. Doob J. Douglas, jr.
A. Grothendieck E. Heinz F. Hirzebruch
E. Hopf W. Maak S. MacLane
W. Magnus M. M. Postnikov F. K. Schmidt
D. S. Scott K. Stein

Geschäftsführende Herausgeber

B. Eckmann B. L. van der Waerden

Anatole Beck

Continuous Flows in the Plane

With the Assistance of
Jonathan and Mirit Lewin

Springer-Verlag
Berlin Heidelberg New York 1974

Professor Anatole Beck
University of Wisconsin, Madison, Wisconsin, USA

Dr. Jonathan and Dr. Mirit Lewin
Ben Gurion University of the Negev, Beer Sheva, Israel

With 47 Figures

AMS Subject Classifications (1970)
Primary 54 H 20, 34 C 35, 54 H 15, 57 E 25
Secondary 54 H 25, 57 E 05, 57 E 20, 58 F 99, 70 G 99

ISBN-13: 978-3-642-65550-0 e-ISBN-13: 978-3-642-65548-7
DOI: 10.1007/978-3-642-65548-7

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to the publisher, the amount of the fee to be determined by agreement with the publisher. © by Springer-Verlag Berlin · Heidelberg 1974. Library of Congress Catalog Card Number 73-11952. Softcover reprint of the hardcover 1st edition 1974

Dedicated to the memory
of my mother

MINNIE BECK
BORNSTEIN

March 4 1904 January 20 1967

She made of her life a gift to those she loved. Without her many sacrifices, this book might never have been written

אשת חיל מי ימצא ורחק מפנינים מכרה.

◆ ◆ ◆

תנו לה מפרי ידיה ויהללוה בשערים מעשיה.

PROLOGUE

Der junge Alexander eroberte Indien

Er allein?

The time of publishing a book is a time to remember and to give thanks. A time to remember all those who by their help have made the book possible and to give thanks for that help. In this case, where the book represents sixteen years of research built on an education of twenty years, the list of those who by their efforts and kindnesses have fostered the education, the research, and the writing of the book runs to many hundreds. Of these, a few dozen have contributed so much that I could not allow the book to go to press without explicit acknowledgement of their assistance.

I begin with my mother, whose contribution over the years was the greatest, and to whose memory this book is dedicated. Widowed at an early age with two young sons, she labored long hours at difficult and unrewarding work to make our educations possible. The price of those educations, to which she contributed unstintingly from her meager earnings, was high, for the accumulated damage to her health led her to an untimely death. Without her support, both financial and psychological, it is questionable whether I would have completed that education, without which this book would have been impossible.

I am deeply conscious of my debt to my teachers, both in school and in the various universities I have attended. Of these, I note especially Walter Prenowitz and Samuel Borofsky of Brooklyn College and Henry Helson, Nelson Dunford, Jacob Schwartz, and Shizuo Kakutani at Yale University. Professor Kakutani, who was my doctoral advisor, never stinted of his time and effort; his kind assistance and demanding discipline initiated me into the mathematical profession.

To Paul Mostert, I give my thanks for introducing me to the field of flows in the plane, and also to Professor Kakutani, Gustav Hedlund, Deane Montgomery, R. H. Bing, and Aryeh Dvoretsky for encouraging my efforts in this area. I thank Professor Hedlund for the administrative

initiative and Doctor Harry Bakwin for the financial substance which together created a fellowship which enabled me to travel and study in Europe at a critical point in my career, and which contributed to my development as an independent mathematician.

My grateful thanks go to my wife, Evelyn, and to my children, Nina and Micah, who gamely endured at second hand many of the frustrations and difficulties of writing this book. Their contribution cannot be overstated.

I cannot thank all the friends whose encouragement fostered my work, but I must give special mention to my brother Bernard, to Donald Newman, Aryeh Dvoretsky, and Konrad Jacobs, and to all my close friends who are my colleagues at Madison.

More important than any in the actual task of writing this book were my students and assistants, Mirit and Jonathan Lewin. Their aid was invaluable in creating this work from the vast pile of notes, published and unpublished articles, jottings, ideas, and results, some correct and some incorrect, some raw and some polished, which represented the material of this book when they joined me in working on it. Reversing the roles of teacher and student, they more than repaid me for my work with them on their doctoral theses by correcting, criticizing, polishing, writing, re-writing, and editing portions of this text. Chapters 4, 5, 8, and 9 are the stuff of those theses, which I am honored to include in this book and proud to exhibit before the mathematical community.

Finally, I wish to give my genuine thanks to the many sources from which I have received financial assistance in my education and my research career: the City and State of New York, Yale University, the National Science Foundation, the Office of Naval Research, Doctor Harry Bakwin, the Wisconsin Alumni Research Foundation, the U.S. Army Mathematics Research Center, the Air Force Office of Scientific Research, the German Academic Exchange Service (DAAD), and the National Research Council.

I do not mean to compare this book to the conquest of India, but it does share this one aspect, that in each case, it is the product of many, many people's work. Although the protocols of the academy will call it my book, I wish here to note and to thank the many, many people who have helped to create it.

ANATOLE BECK

Madison, Wisconsin
9 February 1973

CONTENTS

Index of Symbols XI

Introduction 1

Chapter 1
Elementary Properties of Flows 6
Notes and Remarks to Chapter 1 33

Chapter 2
Special Properties of Plane Flows 34
Notes and Remarks to Chapter 2 57

Chapter 3
Regular and Singular Points 59
Notes and Remarks to Chapter 3 97

Chapter 4
Reparametrization I 100
Notes and Remarks to Chapter 4 135

Chapter 5
Reparametrization II 136
Notes and Remarks to Chapter 5 174

Chapter 6
Existence Theorems I 175
Notes and Remarks to Chapter 6 200

Chapter 7
Existence Theorems II 202
Notes and remarks to Chapter 7 223

Chapter 8
Algebraic Combinations of Flows I 225

Chapter 9
Algebraic Combinations of Flows II 279
Notes and Remarks to Chapters 8 and 9 313

Chapter 10
Fine Structure in $\mathscr{G}_r(\varphi)$ 315

Chapter 11
Fine Structure in $\mathscr{G}_s(\varphi)$ I 348

Chapter 12
Fine Structure in $\mathscr{G}_s(\varphi)$ II 386

Appendix A
Topology 414

Appendix B
The Kurzweil Integral 421

Appendix C
Some Properties of the Plane 429

Epilogue 454

Bibliography 457

Subject Index 459

INDEX OF SYMBOLS

$[a, b]$	434	$\mathrm{inv}(f)$	109	$\mathrm{Prod}^+(\varphi)$	121
(a, b)	434	$\langle J_0, L_0, T\rangle$	46	$\mathrm{Prod}_{\mathfrak{q}}(\varphi)$	121
$a \oplus b$	241	$\mathscr{K}\int_a^b$	421	$\mathrm{Prod}_{\mathfrak{q}}^+(\varphi)$	121
$\alpha_\varphi(x)$	18			$\psi \leq \pi$	223
$\bar{\alpha}_\varphi(x)$	46	$\mathscr{L}(A)$	24	$\psi \oplus \pi$	243
ATI set	88	$\mu \wedge \lambda$	231	$\psi \vee \pi$	244
$[B \text{ side}: A]$	414	$\mu \vee \lambda$	231	$\psi \wedge \pi$	239
$\mathbb{C}$	429	$\mu \perp \lambda$	232	$\psi \mathbin{\mathbf{V}} \pi$	258
$\mathscr{C}(A, B)$	414	$\mu \oplus \lambda$	282	$\psi + \pi$	289
$C_\Omega(f, A)$	441	μ_φ	153	$\psi + \pi$	289
cl	414	$\mathscr{M}(\varphi)$	237	$\mathbb{R}^n$	429
d	429	$\mathscr{M}_{\mathfrak{q}}(\varphi)$	237	$\mathscr{R}(\psi, \pi)$	253
∂	414	$\ominus$	314	$\mathrm{Rep}(\varphi)$	110
$\underline{D}h\ \bar{D}h\ Dh$	22, 23	$N_d(a, \delta)$	430	$\mathrm{Rep}^+(\varphi)$	111
$\mathscr{f}\mathscr{f}$	389	$N_\varrho(a, \delta)$	430	$\mathrm{Rep}_{\mathfrak{q}}(\varphi)$	108
$\mathscr{f}\mathscr{f}_\alpha\ \mathscr{f}\mathscr{f}_\omega$	406	$\omega_\varphi(x)$	19	$\mathrm{Rep}_{\mathfrak{q}}^+(\varphi)$	108
$f[\varphi]$	18	$\bar{\omega}_\varphi(x)$	46	ϱ	430
$\mathscr{F}(\varphi)$	7	$\mathbf{1}$	10	$\mathscr{S}(\varphi)$	20
$\mathscr{F}_r(\varrho)$	57	$\mathbf{0}(\varphi)$	6	$\mathbf{S}^n$	429
$\mathscr{F}_s(\varphi)$	57	$\mathscr{O}_\varphi(x)$	6	$\mathbf{\Sigma}(\varphi)$	153
f rep φ	105	$\mathrm{outs}(J)$	436	$\langle t_1, t_2, x_1, x_2, J_0, L_0, T\rangle$	46
Γ_δ	430	$\mathrm{p}_\varphi(x)$	8	T-set	76
$\mathscr{G}(\varphi)$	7	$\dot{\varphi}(x)$	22	T_σ-set	81
$\mathscr{G}_r(\varphi)$	60	$\lvert\dot{\varphi}\rvert(x)$	22	$u\varphi$	120
$\mathscr{G}_s(\varphi)$	60	$\lvert\dot{\varphi}\rvert_d$	177	$\bigcup_{\alpha\in A} \varphi_\alpha$	176
ind_γ	450, 451	$\varphi \equiv \psi$	336	$V_h([a, b])$	24
$\mathrm{ins}(J)$	436	$\varphi \equiv^+ \psi$	336	WATI set	88
int	414	$\mathrm{Prod}(\varphi)$	121	$X^{(\alpha)}$	203