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To our wives Sharleen and Shelly

Preface

This monograph evolved over the past five years. It had its origin as a set of lecture notes prepared for the Ninth Summer School of Mathematical Physics held at Ravello, Italy, in 1984 and was further refined in seminars and lectures given primarily at the University of Colorado.

The material presented is the product of a single mathematical question raised by Dave Kassoy over ten years ago. This question and its partial resolution led to a successful, exciting, almost unique interdisciplinary collaborative scientific effort.

The mathematical models described are often times deceptively simple in appearance. But they exhibit a mathematical richness and beauty that belies that simplicity and affirms their physical significance. The mathematical tools required to resolve the various problems raised are diverse, and no systematic attempt is made to give the necessary mathematical background. The unifying theme of the monograph is the set of models themselves.

This monograph would never have come to fruition without the enthusiasm and drive of Dave Eberly–a former student, now collaborator and coauthor–and without several significant breakthroughs in our understanding of the phenomena of blowup or thermal runaway which certain models discussed possess.

A collaborator and former student who has made significant contributions throughout is Alberto Bressan. There are many other collaborators–William Troy, Watson Fulks, Andrew Lacey, Klaus Schmitt–and former students–Paul Talaga and Richard Ely–who must be acknowledged and thanked.

Finally, I would like to acknowledge the continued support of the Army Research Office and its director, Jagdish Chandra.

October 1988

Jerrold Bebernes
University of Colorado

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