

# AN INTRODUCTION TO FLUID DYNAMICS

BY

G. K. BATCHELOR, F.R.S.

*Professor of Applied Mathematics in the University of Cambridge*



## CONTENTS

<i>Preface</i>	<i>page</i> xiii
<i>Conventions and Notation</i>	xviii
<b>Chapter 1. The Physical Properties of Fluids</b>	
1.1 Solids, liquids and gases	1
1.2 The continuum hypothesis	4
1.3 Volume forces and surface forces acting on a fluid	7
Representation of surface forces by the stress tensor, 9	
The stress tensor in a fluid at rest, 12	
1.4 Mechanical equilibrium of a fluid	14
A body 'floating' in fluid at rest, 16	
Fluid at rest under gravity, 18	
1.5 Classical thermodynamics	20
1.6 Transport phenomena	28
The linear relation between flux and the gradient of a scalar intensity, 30	
The equations for diffusion and heat conduction in isotropic media at rest, 32	
Molecular transport of momentum in a fluid, 36	
1.7 The distinctive properties of gases	37
A perfect gas in equilibrium, 38	
Departures from the perfect-gas laws, 45	
Transport coefficients in a perfect gas, 47	
Other manifestations of departure from equilibrium of a perfect gas, 50	
1.8 The distinctive properties of liquids	53
Equilibrium properties, 55	
Transport coefficients, 57	
1.9 Conditions at a boundary between two media	60
Surface tension, 60	
Equilibrium shape of a boundary between two stationary fluids, 63	
Transition relations at a material boundary, 68	
<b>Chapter 2. Kinematics of the Flow Field</b>	
2.1 Specification of the flow field	71
Differentiation following the motion of the fluid, 72	
2.2 Conservation of mass	73
Use of a stream function to satisfy the mass-conservation equation, 75	
2.3 Analysis of the relative motion near a point	79
Simple shearing motion, 83	

2.4	Expression for the velocity distribution with specified rate of expansion and vorticity	page 84
2.5	Singularities in the rate of expansion. Sources and sinks	88
2.6	The vorticity distribution	92
	Line vortices, 93	
	Sheet vortices, 96	
2.7	Velocity distributions with zero rate of expansion and zero vorticity	99
	Conditions for $\nabla\phi$ to be determined uniquely, 102	
	Irrotational solenoidal flow near a stagnation point, 105	
	The complex potential for irrotational solenoidal flow in two dimensions, 106	
2.8	Irrotational solenoidal flow in doubly-connected regions of space	108
	Conditions for $\nabla\phi$ to be determined uniquely, 112	
2.9	Three-dimensional flow fields extending to infinity	114
	Asymptotic expressions for $\mathbf{u}_s$ and $\mathbf{u}_v$ , 114	
	The behaviour of $\phi$ at large distances, 117	
	Conditions for $\nabla\phi$ to be determined uniquely, 119	
	The expression of $\phi$ as a power series, 120	
	Irrotational solenoidal flow due to a rigid body in translational motion, 122	
2.10	Two-dimensional flow fields extending to infinity	124
	Irrotational solenoidal flow due to a rigid body in translational motion, 128	
<b>Chapter 3. Equations Governing the Motion of a Fluid</b>		
3.1	Material integrals in a moving fluid	131
	Rates of change of material integrals, 133	
	Conservation laws for a fluid in motion, 135	
3.2	The equation of motion	137
	Use of the momentum equation in integral form, 138	
	Equation of motion relative to moving axes, 139	
3.3	The expression for the stress tensor	141
	Mechanical definition of pressure in a moving fluid, 141	
	The relation between deviatoric stress and rate-of-strain for a Newtonian fluid, 142	
	The Navier–Stokes equation, 147	
	Conditions on the velocity and stress at a material boundary, 148	
3.4	Changes in the internal energy of a fluid in motion	151
3.5	Bernoulli's theorem for steady flow of a frictionless non-conducting fluid	156
	Special forms of Bernoulli's theorem, 161	
	Constancy of $H$ across a transition region in one-dimensional steady flow, 163	
3.6	The complete set of equations governing fluid flow	164
	Isentropic flow, 165	
	Conditions for the velocity distribution to be approximately solenoidal, 167	
3.7	Concluding remarks to chapters 1, 2 and 3	171

## Chapter 4. Flow of a Uniform Incompressible Viscous Fluid

4.1	Introduction	<i>page</i> 174
	Modification of the pressure to allow for the effect of the body force, 176	
4.2	Steady unidirectional flow	179
	Poiseuille flow, 180	
	Tubes of non-circular cross-section, 182	
	Two-dimensional flow, 182	
	A model of a paint-brush, 183	
	A remark on stability, 185	
4.3	Unsteady unidirectional flow	186
	The smoothing-out of a discontinuity in velocity at a plane, 187	
	Plane boundary moved suddenly in a fluid at rest, 189	
	One rigid boundary moved suddenly and one held stationary, 190	
	Flow due to an oscillating plane boundary, 191	
	Starting flow in a pipe, 193	
4.4	The Ekman layer at a boundary in a rotating fluid	195
	The layer at a free surface, 197	
	The layer at a rigid plane boundary, 199	
4.5	Flow with circular streamlines	201
4.6	The steady jet from a point source of momentum	205
4.7	Dynamical similarity and the Reynolds number	211
	Other dimensionless parameters having dynamical significance, 215	
4.8	Flow fields in which inertia forces are negligible	216
	Flow in slowly-varying channels, 217	
	Lubrication theory, 219	
	The Hele-Shaw cell, 222	
	Percolation through porous media, 223	
	Two-dimensional flow in a corner, 224	
	Uniqueness and minimum dissipation theorems, 227	
4.9	Flow due to a moving body at small Reynolds number	229
	A rigid sphere, 230	
	A spherical drop of a different fluid, 235	
	A body of arbitrary shape, 238	
4.10	Oseen's improvement of the equation for flow due to moving bodies at small Reynolds number	240
	A rigid sphere, 241	
	A rigid circular cylinder, 244	
4.11	The viscosity of a dilute suspension of small particles	246
	The flow due to a sphere embedded in a pure straining motion, 248	
	The increased rate of dissipation in an incompressible suspension, 250	
	The effective expansion viscosity of a liquid containing gas bubbles, 253	
4.12	Changes in the flow due to moving bodies as $R$ increases from 1 to about 100	255

<b>Chapter 5. Flow at Large Reynolds Number: Effects of Viscosity</b>	
<b>5.1</b>	<b>Introduction</b> <span style="float: right;"><i>page</i> 264</span>
<b>5.2</b>	<b>Vorticity dynamics</b> <span style="float: right;">266</span>
	The intensification of vorticity by extension of vortex-lines, 270
<b>5.3</b>	<b>Kelvin's circulation theorem and vorticity laws for an inviscid fluid</b> <span style="float: right;">273</span>
	The persistence of irrotationality, 276
<b>5.4</b>	<b>The source of vorticity in motions generated from rest</b> <span style="float: right;">277</span>
<b>5.5</b>	<b>Steady flows in which vorticity generated at a solid surface is prevented by convection from diffusing far away from it</b> <span style="float: right;">282</span>
	(a) Flow along plane and circular walls with suction through the wall, 282
	(b) Flow toward a 'stagnation point' at a rigid boundary, 285
	(c) Centrifugal flow due to a rotating disk, 290
<b>5.6</b>	<b>Steady two-dimensional flow in a converging or diverging channel</b> <span style="float: right;">294</span>
	Purely convergent flow, 297
	Purely divergent flow, 298
	Solutions showing both outflow and inflow, 301
<b>5.7</b>	<b>Boundary layers</b> <span style="float: right;">302</span>
<b>5.8</b>	<b>The boundary layer on a flat plate</b> <span style="float: right;">308</span>
<b>5.9</b>	<b>The effects of acceleration and deceleration of the external stream</b> <span style="float: right;">314</span>
	The similarity solution for an external stream velocity proportional to $x^m$ , 316
	Calculation of the steady boundary layer on a body moving through fluid, 318
	Growth of the boundary layer in initially irrotational flow, 321
<b>5.10</b>	<b>Separation of the boundary layer</b> <span style="float: right;">325</span>
<b>5.11</b>	<b>The flow due to bodies moving steadily through fluid</b> <span style="float: right;">331</span>
	Flow without separation, 332
	Flow with separation, 337
<b>5.12</b>	<b>Jets, free shear layers and wakes</b> <span style="float: right;">343</span>
	Narrow jets, 343
	Free shear layers, 346
	Wakes, 348
<b>5.13</b>	<b>Oscillatory boundary layers</b> <span style="float: right;">353</span>
	The damping force on an oscillating body, 355
	Steady streaming due to an oscillatory boundary layer, 358
	Applications of the theory of steady streaming, 361

5.14	Flow systems with a free surface	page 364
	The boundary layer at a free surface, 364	
	The drag on a spherical gas bubble rising steadily through liquid, 367	
	The attenuation of gravity waves, 370	
5.15	Examples of use of the momentum theorem	372
	The force on a regular array of bodies in a stream, 372	
	The effect of a sudden enlargement of a pipe, 373	
<b>Chapter 6. Irrotational Flow Theory and its Applications</b>		
6.1	The role of the theory of flow of an inviscid fluid	378
6.2	General properties of irrotational flow	380
	Integration of the equation of motion, 382	
	Expressions for the kinetic energy in terms of surface integrals, 383	
	Kelvin's minimum energy theorem, 384	
	Positions of a maximum of $q$ and a minimum of $p$ , 384	
	Local variation of the velocity magnitude, 386	
6.3	Steady flow: some applications of Bernoulli's theorem and the momentum theorem	386
	Efflux from a circular orifice in an open vessel, 387	
	Flow over a weir, 391	
	Jet of liquid impinging on a plane wall, 392	
	Irrotational flow which may be made steady by choice of rotating axes, 396	
6.4	General features of irrotational flow due to a moving rigid body	398
	The velocity at large distances from the body, 399	
	The kinetic energy of the fluid, 402	
	The force on a body in translational motion, 404	
	The acceleration reaction, 407	
	The force on a body in accelerating fluid, 409	
6.5	Use of the complex potential for irrotational flow in two dimensions	409
	Flow fields obtained by special choice of the function $w(z)$ , 410	
	Conformal transformation of the plane of flow, 413	
	Transformation of a boundary into an infinite straight line, 418	
	Transformation of a closed boundary into a circle, 420	
	The circle theorem, 422	
6.6	Two-dimensional irrotational flow due to a moving cylinder with circulation	423
	A circular cylinder, 424	
	An elliptic cylinder in translational motion, 427	
	The force and moment on a cylinder in steady translational motion, 433	
6.7	Two-dimensional aerofoils	435
	The practical requirements of aerofoils, 435	
	The generation of circulation round an aerofoil and the basis for Joukowski's hypothesis, 438	
	Aerofoils obtained by transformation of a circle, 441	
	Joukowski aerofoils, 444	

6.8	Axisymmetric irrotational flow due to moving bodies	page 449
	Generalities, 449	
	A moving sphere, 452	
	Ellipsoids of revolution, 455	
	Body shapes obtained from source singularities on the axis of symmetry, 458	
	Semi-infinite bodies, 460	
6.9	Approximate results for slender bodies	463
	Slender bodies of revolution, 463	
	Slender bodies in two dimensions, 466	
	Thin aerofoils in two dimensions, 467	
6.10	Impulsive motion of a fluid	471
	Impact of a body on a free surface of liquid, 473	
6.11	Large gas bubbles in liquid	474
	A spherical-cap bubble rising through liquid under gravity, 475	
	A bubble rising in a vertical tube, 477	
	A spherical expanding bubble, 479	
6.12	Cavitation in a liquid	481
	Examples of cavity formation in steady flow, 482	
	Examples of cavity formation in unsteady flow, 485	
	Collapse of a transient cavity, 486	
	Steady-state cavities, 491	
6.13	Free-streamline theory, and steady jets and cavities	493
	Jet emerging from an orifice in two dimensions, 495	
	Two-dimensional flow past a flat plate with a cavity at ambient pressure, 497	
	Steady-state cavities attached to bodies held in a stream of liquid, 502	
 <b>Chapter 7. Flow of Effectively Inviscid Fluid with Vorticity</b>		
7.1	Introduction	507
	The self-induced movement of a line vortex, 509	
	The instability of a sheet vortex, 511	
7.2	Flow in unbounded fluid at rest at infinity	517
	The resultant force impulse required to generate the motion, 518	
	The total kinetic energy of the fluid, 520	
	Flow with circular vortex-lines, 521	
	Vortex rings, 522	
7.3	Two-dimensional flow in unbounded fluid at rest at infinity	527
	Integral invariants of the vorticity distribution, 528	
	Motion of a group of point vortices, 530	
	Steady motions, 532	
7.4	Steady two-dimensional flow with vorticity throughout the fluid	536
	Uniform vorticity in a region bounded externally, 538	
	Fluid in rigid rotation at infinity, 539	
	Fluid in simple shearing motion at infinity, 541	

7.5	Steady axisymmetric flow with swirl The effect of a change of cross-section of a tube on a stream of rotating fluid, 546 The effect of a change of external velocity on an isolated vortex, 550	<i>page</i> 543
7.6	Flow systems rotating as a whole The restoring effect of Coriolis forces, 555 Steady flow at small Rossby number, 557 Propagation of waves in a rotating fluid, 559 Flow due to a body moving along the axis of rotation, 564	555
7.7	Motion in a thin layer on a rotating sphere Geostrophic flow, 571 Flow over uneven ground, 573 Planetary waves, 577	567
7.8	The vortex system of a wing General features of the flow past lifting bodies in three dimensions, 580 Wings of large aspect ratio, and 'lifting-line' theory, 583 The trailing vortex system far downstream, 589 Highly swept wings, 591	580

### Appendices

1	Measured values of some physical properties of common fluids (a) Dry air at a pressure of one atmosphere, 594 (b) The Standard Atmosphere, 595 (c) Pure water, 595 (d) Diffusivities for momentum and heat at 15 °C and 1 atm, 597 (e) Surface tension between two fluids, 597	594
2	Expressions for some common vector differential quantities in orthogonal curvilinear co-ordinate systems	598
	<i>Publications referred to in the text</i>	604
	<i>Subject Index</i>	609

Plates 1 to 24 are between pages 364 and 365