Zdzisław Brzeźniak and Tomasz Zastawniak

Basic Stochastic Processes

A Course Through Exercises

With 21 Figures

Springer

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