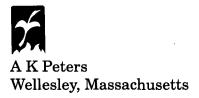
Set Theory

On the Structure of the Real Line

Tomek Bartoszyński Department of Mathematics Boise State University Boise, Idaho

Haim Judah Department of Mathmatics Bar Ilan University Ramat Gan, Israel



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