

Lecture Notes in Mathematics

A collection of informal reports and seminars

Edited by A. Dold, Heidelberg and B. Eckmann, Zürich

Series: Forschungsinstitut für Mathematik, ETH Zürich

236

Michael Barr
Pierre A. Grillet
Donovan H. van Osdol

Exact Categories and
Categories of Sheaves



Springer-Verlag

Berlin · Heidelberg · New York 1971

Michael Barr

University of Fribourg, Fribourg/Switzerland and
McGill University, Montreal/Canada

Pierre A. Grillet

Kansas State University, Manhattan, KS/USA

Donovan H. van Osdol

University of New Hampshire, Durham, NH/USA

AMS Subject Classifications (1970): Primary: 18B15, 18D99, 18F20
Secondary: 18A25, 18C10

ISBN 3-540-05678-5 Springer-Verlag Berlin · Heidelberg · New York
ISBN 0-387-05678-5 Springer-Verlag New York · Heidelberg · Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks.

Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to the publisher, the amount of the fee to be determined by agreement with the publisher.

© by Springer-Verlag Berlin · Heidelberg 1971. Library of Congress Catalog Card Number 72-180690. Printed in Germany.

Offsetdruck: Julius Beltz, Hemsbach/Bergstr.

Table of Contents

Michael Barr: Exact Categories

Introduction	1
------------------------	---

Chapter I. The Elementary Theory

1. Definitions and examples	4
2. Preliminary results.	7
3. Additive exact categories.	12
4. Regular epimorphism sheaves.	15
5. Constructions on regular and exact categories.	18

Chapter II. Locally Presentable Categories

1. Definitions.	27
2. Preliminary results.	30
3. Rank.	33
4. Kan extension of functors.	37
5. Toposes.	42

Chapter III. The Embedding

1. Statement of results	44
2. Support.	47
3. Diagrams	52
4. The Lubkin completion process.	57
5. The embedding.	62
6. Diagram chasing.	69

Chapter IV. Groups and Representations

1. Preliminaries.	75
2. Tensor products.	79
3. Principal objects.	84
4. Structure of groups.	87

Chapter V. Cohomology

1. Definitions.	90
2. The exact sequence.	94
3. Abelian groups.	98
4. Extensions.	102
Appendix: The Giraud Theorems (Characterizing toposes) . .	106
REFERENCES	119

Pierre Antoine Grillet: Regular Categories

Introduction.	121
I. Examples and elementary properties	124
1. Decompositions.	125
2. Regular categories: definition and examples	134
3. Subobjects; direct and inverse images.	138
4. Relations	143
5. Congruences	154
6. Limits and colimits in a regular category	162
Synopsis of definitions and formulae	168
II. Directed colimits in regular categories	170
1. The main theorem: direct part	172
2. Converse: preservation of monomorphisms	176
3. Converse: preservation of finite limits	182
4. Additional properties of directed colimits.	184
III. Sheaves in regular categories.	191
1. Grothendieck topologies and sheaves	192
2. The Heller and Rowe construction of the associated sheaf	195
3. The case of a \mathcal{C}_4 regular category	205
4. Stalk properties.	217
REFERENCES	221

Donovan H. van Osdol: Sheaves in Regular Categories

Introduction.	223
I. Transfer Theoremes for Triples.	223
II. Transfer Theoremes for Cotriples.	226
III. Sheaves	230
IV. Interpretation and Examples	235
REFERENCES	239