

Contents

Instructor Preface	ix
Student Preface	xiii
Acknowledgments	xix
I. Foundations of Geometry in the Plane	
I.1. The Real Numbers	1
I.2. The Incidence Axioms	6
I.3. Distance and the Ruler Axiom	17
I.4. Betweenness	22
I.5. The Plane Separation Axiom	27
I.6. The Angular Measure Axioms	34
I.7. Triangles and the SAS Axiom	46
I.8. Geometric Inequalities	56
I.9. Parallelism	62
I.10. The Parallel Postulate	70
I.11. Directed Angle Measure and Ray Translation	84
I.12. Similarity	94
I.13. Circles	110
I.14. Bolzano's Theorem	115
I.15. Axioms for the Euclidean Plane	119
II. Isometries in the Plane: Products of Reflections	
II.1. Transformations in the Plane	121
II.2. Isometries in the Plane	135
II.3. Composition and Inversion	146
II.4. Fixed Points and the First Structure Theorem	156
II.5. Triangle Congruence and Isometries	161
III. Isometries in the Plane: Classification and Structure	
III.1. Two Reflections: Translations and Rotations	165
III.2. Glide Reflections	181
III.3. The Classification Theorem	188
III.4. Orientation	191
III.5. Groups of Transformations	199
III.6. The Second Structure Theorem	206
III.7. Rotation Angles	211

IV. Similarities in the Plane	
IV.1. Elementary Properties of Similarities	217
IV.2. Dilations as Similarities	224
IV.3. The Structure of Similarities	231
IV.4. Orientation and Rotation Angles	235
IV.5. Fixed Points for Similarities	240
V. Conjugacy and Geometric Equivalence	
V.1. Congruence and Geometric Equivalence	251
V.2. Geometric Equivalence of Transformations: Conjugacy	256
V.3. Geometric Equivalence under Similarities	266
V.4. Euclidean Geometry Derived from Transformations	276
VI. Applications to Plane Geometry	
VI.1. Symmetry in Early Geometry	287
VI.2. The Classical Coincidences	292
VI.3. Dilation by Minus Two around the Centroid	298
VI.4. Reflections, Light, and Distance	309
VI.5. Fagnano's Problem and the Orthic Triangle	315
VI.6. The Fermat Problem	322
VI.7. The Circle of Apollonius	340
VII. Symmetric Figures in the Plane	
VII.1. Symmetry Groups	347
VII.2. Invariant Sets and Orbits	356
VII.3. Bounded Figures in the Plane	363
VIII. Frieze and Wallpaper Groups	
VIII.1. Point Groups and Translation Subgroups	376
VIII.2. Frieze Groups	399
VIII.3. Two-Dimensional Translation Lattices	416
VIII.4. Wallpaper Groups	439
IX. Area, Volume, and Scaling	
IX.1. Length of Curves	459
IX.2. Area of Polygonal Regions: Basic Properties	467
IX.3. Area and Equidecomposability	482
IX.4. Area by Approximation	487
IX.5. Area and Similarity	505
IX.6. Scaling and Dimension	520
References	531
Index	533