Edward J. Barbeau

Pell's Equation



Edward J. Barbeau Department of Mathematics University of Toronto 100 St. George Street Toronto, Ontario M5S 3G3 Canada barbeau@math.toronto.edu

Series Editors: Katalin A. Bencsáth Mathematics School of Science Manhattan College Riverdale, NY 10471 USA katalin.benscath@manhattan.edu

Paul R. Halmos Department of Mathematics Santa Clara University Santa Clara, CA 95053 USA phalmos@scuacc.scu.edu

With 9 illustrations.

Mathematics Subject Classification (2000): 26-06, 26Bxx, 34Axx, 30Axx

Library of Congress Cataloging-in-Publication Data Barbeau, Edward, 1938– Pell's equation / Edward J. Barbeau. p. cm. — (Problem books in mathematics) Includes bibliographical references and index. ISBN 978-1-4419-3040-8 ISBN 978-0-387-22602-6 (eBook) DOI 10.1007/978-0-387-22602-6 1. Pell's equation. 2. Number theory. I. Title. II. Series. QA242.B4395 2002 512'.7—dc21 2002070735

Printed on acid-free paper.

© 2003 Springer-Verlag New York, Inc.

Softcover reprint of the hardcover 1st edition 2003

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden. The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

9 8 7 6 5 4 3 2 1 SPIN 10883222

www.springer-ny.com

Springer-Verlag New York Berlin Heidelberg A member of BertelsmannSpringer Science+Business Media GmbH

Contents

Preface	vii
1. The Square Root of 2	1
2. Problems Leading to Pell's Equation and Preliminary Investigations	16
3. Quadratic Surds	32
4. The Fundamental Solution	43
5. Tracking Down the Fundamental Solution	55
6. Pell's Equation and Pythagorean Triples	81
7. The Cubic Analogue of Pell's Equation	92
8. Analogues of the Fourth and Higher Degrees	113
9. A Finite Version of Pell's Equation	126
Answers and Solutions	139
Glossary	201
References	205
Index	211