Edward J. Barbeau

## Pell's Equation

Edward J. Barbeau
Department of Mathematics
University of Toronto
100 St. George Street
Toronto, Ontario M5S 3G3
Canada
barbeau@math.toronto.edu

Series Editors:
Katalin A. Bencsáth
Mathematics
School of Science
Manhattan College
Riverdale, NY 10471
USA
katalin.benscath@manhattan.edu

Paul R. Halmos<br>Department of Mathematics<br>Santa Clara University<br>Santa Clara, CA 95053<br>USA<br>phalmos@scuacc.scu.edu

With 9 illustrations.

Mathematics Subject Classification (2000): 26-06, 26Bxx, 34Axx, 30Axx
Library of Congress Cataloging-in-Publication Data
Barbeau, Edward, 1938-
Pell's equation / Edward J. Barbeau.
p. cm. - (Problem books in mathematics)

Includes bibliographical references and index.
ISBN 978-1-4419-3040-8 ISBN 978-0-387-22602-6 (eBook)
DOI 10.1007/978-0-387-22602-6

1. Pell's equation. 2. Number theory. I. Title. II. Series.

QA242 .B4395 2002
512'.7-dc21
2002070735
Printed on acid-free paper.

## © 2003 Springer-Verlag New York, Inc.

Softcover reprint of the hardcover 1st edition 2003
All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden. The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

## Contents

Preface ..... vii

1. The Square Root of 2 ..... 1
2. Problems Leading to Pell's Equation and Preliminary Investigations ..... 16
3. Quadratic Surds ..... 32
4. The Fundamental Solution ..... 43
5. Tracking Down the Fundamental Solution ..... 55
6. Pell's Equation and Pythagorean Triples ..... 81
7. The Cubic Analogue of Pell's Equation ..... 92
8. Analogues of the Fourth and Higher Degrees ..... 113
9. A Finite Version of Pell's Equation ..... 126
Answers and Solutions ..... 139
Glossary ..... 201
References ..... 205
Index ..... 211
