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(continued after index)

Thomas Banchoff
John Wermer

Linear Algebra Through Geometry

Second Edition

With 92 Illustrations

Springer

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To our wives Lynore and Kerstin

Preface to the Second Edition

In this book we lead the student to an understanding of elementary linear algebra by emphasizing the geometric significance of the subject.

Our experience in teaching undergraduates over the years has convinced us that students learn the new ideas of linear algebra best when these ideas are grounded in the familiar geometry of two and three dimensions. Many important notions of linear algebra already occur in these dimensions in a non-trivial way, and a student with a confident grasp of the ideas will encounter little difficulty in extending them to higher dimensions and to more abstract algebraic systems. Moreover, we feel that this geometric approach provides a solid basis for the linear algebra needed in engineering, physics, biology, and chemistry, as well as in economics and statistics.

The great advantage of beginning with a thorough study of the linear algebra of the plane is that students are introduced quickly to the most important new concepts while they are still on the familiar ground of two-dimensional geometry. In short order, the student sees and uses the notions of dot product, linear transformations, determinants, eigenvalues, and quadratic forms. This is done in Chapters 2.0–2.7.

Then, the very same outline is used in Chapters 3.0–3.7 to present the linear algebra of three-dimensional space, so that the former ideas are reinforced while new concepts are being introduced.

In Chapters 4.0–4.2, we deal with geometry in $\mathbb{R}^n$ for $n \geq 4$. We introduce linear transformations and matrices in $\mathbb{R}^4$, and we point out that the step from $\mathbb{R}^4$ to $\mathbb{R}^n$ with $n > 4$ is now almost immediate. In Chapters 4.3 and 4.4, we treat systems of linear equations in n variables.

In the present edition, we have added Chapter 5 on vector spaces, Chapter 6 on inner products on a vector space, and Chapter 7 on

symmetric $n \times n$ matrices and quadratic forms in n variables. Finally, in Chapter 8 we deal with three applications:

(1) differential systems, that is, systems of linear first-order differential equations;
(2) least-squares method in data analysis; and
(3) curvature of surfaces in $\mathbb{R}^3$, which are given as graphs of functions of two variables.

Except for Chapter 8, the student need only know basic high-school algebra and geometry and introductory trigonometry in order to read this book. In fact, we believe that high-school seniors who are interested in mathematics could read much of this book on their own. To read Chapter 8, students should have a knowledge of elementary calculus.

Acknowledgments

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