Asymptotic Approaches in Nonlinear Dynamics

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An ever increasing number of scientific disciplines deal with complex systems. These are systems that are composed of many parts which interact with one another in a more or less complicated manner. One of the most striking features of many such systems is their ability to spontaneously form spatial or temporal structures. A great variety of these structures are found, in both the inanimate and the living world. In the inanimate world of physics and chemistry, examples include the growth of crystals, coherent oscillations of laser light, and the spiral structures formed in fluids and chemical reactions. In biology we encounter the growth of plants and animals (morphogenesis) and the evolution of species. In medicine we observe, for instance, the electromagnetic activity of the brain with its pronounced spatio-temporal structures. Psychology deals with characteristic features of human behavior ranging from simple pattern recognition tasks to complex patterns of social behavior. Examples from sociology include the formation of public opinion and cooperation or competition between social groups.

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New Trends and Applications

With 58 Figures

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Preface

> How well is Nature simulated by the varied asymptotic models that imaginative scientists have invented?
>
> B. Birkhoff [52]

This book deals with asymptotic methods in nonlinear dynamics. For the first time a detailed and systematic treatment of new asymptotic methods in combination with the Padé approximant method is presented.

Most of the basic results included in this manuscript have not been treated but just mentioned in the literature. Providing a state-of-the-art review of asymptotic applications, this book will prove useful as an introduction to the field for novices as well a reference for specialists.

Asymptotic methods of solving mechanical and physical problems have been developed by many authors. For example, we can refer to the excellent courses by A. Nayfeh [119–122], M. Van Dyke [154], E.J. Hinch [94] and many others [59, 66, 95, 109, 126, 155, 163, 50d, 59d]. The main features of the monograph presented are: 1) it is devoted to the basic principles of asymptotics and its applications, and 2) it deals with both traditional approaches (such as regular and singular perturbations, averaging and homogenization, perturbations of the domain and boundary shape) and less widely used, new approaches such as one- and two-point Padé approximants, the distributional approach, and the method of boundary perturbations.

Many results are reported in English for the first time. The choice of topics reflects the authors' research experience and involvement in industrial applications. The authors hope that this book will introduce the reader to the field of asymptotic simplification of the problems of the theory of oscillations, and will be useful as a handbook of methods of asymptotic integration as well.

The narration is commonly based on examples given by applied mechanics of structures (primarily, plates and shells) and fluid mechanics, but scarcely of quantum physics. Obviously, the methods in question are really versatile in application, covering applied mathematics, physics, mechanics and other basic sciences. The authors have paid special attention to examples and discussion of results rather than to burying the ideas in formalism, notation, and technical details. The aim is to introduce mathematicians – as well as physicists, engineers, and other consumers of asymptotic methods – to the world of ideas and methods in this burgeoning area.

The effect of asymptotic methods (AM) on the theory of oscillations increases multifold. The vitality and prospect of AM becomes obvious from the fact that active interaction between numerical and analytical methods is accomplished via asymptotics. It is a pity that asymptotic mathematics does not occupy the decent place in education programmes of high schools. Certain tutorial aspects, useful for training mechanics, physicists, applied mathematicians and engineers, are presented here.

Let us scan in detail the contents of the chapters.

An introduction the depicting the principal ideas of asymptotic approaches through simple, "transparent" examples is given.

The first part is devoted to discrete systems.

First, an introduction to classical perturbation techniques are presented. The KBM methods and the equivalent linearization are described in some detail. Nonconservative nonautonomous systems are considered and nonresonance oscillations as well as oscillations in the neighbourhood of resonance are analysed. A general approach to the analysis of unstationary nonlinear systems is given. Particular attention is paid to consideration of combined parametric and self-excited oscillations in a three-degree-of-freedom mechanical system. This example includes a derivation of the equation of motion and a determination of instability zones. The so-called modified Poincaré approach is presented and illustrated on the basis of a one-degree-of-freedom system and then this approach is extended to the analysis of general nonlinear systems. Then, the Hopf bifurcation is discussed from the viewpoint of the asymptotic approach. Finally, a method of controlling and improving the stability of periodic orbits of vibro-impact systems is proposed. This method is based on the feedback loop control with a time delay. This subchapter includes two parts of our investigation. In what follows a perturbation technique is applied to estimate delay loop coefficients for the improvement of stability of the vibro-impact motion for one-degree-of-fredom systems. Then, control of the periodic motion of the one-degree-of-freedom vibro-impact oscillator is analysed numerically, showing good agreement with the analytical prediction.

Nonlinear normal vibrations are a generalization of normal (principal) vibrations of linear systems. In the normal mode all position coordinates can be defined from any one of them. Using normal modes of nonlinear systems gives very interesting results, and in Sect. 2.10 we write about some aspects of the asymptotic construction of an object.

Progress in the applications of AM in the theory of oscillations as well as in applied mathematics on the whole is closely linked with the introduction of new small parameters and, respectively, new asymptotic procedures. This is the field of Sect. 2.11.

In Sect. 2.12 we deal with one- and two-point Padé approximants (PA). Usually PAs are used for the extension of the area of applicability of perturbation series. We propose to use PAs in connection with AM in many new cases, in particular:

1. Estimation of the convergence domain for perturbation series. In particular, such estimation may be obtained on the basis of the comparison of the perturbation series and PA. This result is justified by many interesting examples from nonlinear mechanics.
2. Elimination of nonuniformities of asymptotic expansions. The PA eliminates nonuniformities of asymptotic expansions in more important mechanical problems in a simpler way than, for instance, Lighthill's method. Up till now two-point PAs (TPPAs) have not been so widely used in mechanics. We represent new applications of the TPPAs for matching local expansions in nonlinear dynamics.

The second part specifies the most important and useful forms and techniques of asymptotic thinking for the theory of oscillations for continual systems.

Relations between the dynamics of discrete and continual systems are based on the procedures of discretization and continualization. The procedure of discretization is described well in many books, so we have paid some attention only to the continualization (the passage from discrete to continuous systems) in Sect. 3.1.

Section 3.2 is devoted to the homogenization approach. The main problem in this field is in the solution of the so-called cell (or local) problem. This problem has usually been treated by a numerical method. We have used an asymptotic method for solving the cell problem and have constructed an approach in this book. The approach presented fills the substantial gap between numerical methods of the thin shell theory, which lack generality and the possibility of grasping the common features of the behaviour of the structures concerned, and approximate schemes, based on heuristic hypotheses. The methods proposed are wide-ranging in applications and lead to simple and clear design formulae, useful for practical analyses. The averaging approach is one of the most useful tools in the theory of nonlinear oscillations. Usually it is used with respect to time variables, but in Sect. 3.3 we show new perspectives for averaging with respect to spatial variables.

V.V. Bolotin proposed an effective asymptotic method for the investigation of linear continuous elastic system oscillations with complicated boundary conditions. The main idea of this approach is in the separation of the continuous elastic system into two parts. Then the matching procedure permits us to obtain a complete solution of the dynamics problem in a relatively simple form. The idea of Bolotin's asymptotic method was generelized for the nonlinear case in Sect. 3.4.

Regular and singular asymptotics in a wide range of forms are the old, but formidable weapons in the armoury of an asymptotic mathematician. In Sect. 3.5 a lot of interesting problems are solved on this basis, and some interesting aspects of the application of these traditional approaches are notified.

A new AM for solving mixed boundary value problems is considered in Sect. 3.6. The parameter ε is introduced into the boundary conditions in such a way that the $\varepsilon = 0$ case corresponds to the simple boundary problems and the case $\varepsilon = 1$ corresponds to the general problem under consideration. Then, the ε-expansion of the solution is obtained. As a rule, the expansion of the solution is divergent just at the point $\varepsilon = 1$. The PAs are used to remove this divergence.

The TPPA in application to nonlinear dynamic problems for a continuous system – a plate on a nonlinear foundation – is displayed in Sect. 3.7.

The discovery of the soliton in 1965 by Kruscal and Zabusky has brought revolutionary changes in nonlinear science, and we describe some uses of the soliton technique in Sect. 3.8.

In Sect. 3.9 a nonlinear analysis of spatial structures is described on the basis of the so-called modified envelope equation.

The third part of the book includes an investigation of discrete-continuous systems. In Sect. 4.1, periodic oscillations of discrete-continuous systems with a time delay are analysed. In Sect. 4.2 a simple perturbation technique is described as it is used in the analysis of discrete-continuous systems with a time delay and with homogeneous boundary conditions. In Sect. 4.3 the nonlinear behaviour of an electromechanical system is investigated on the basis of an averaging technique supported by symbolic computation using the Mathematica package. Then the obtained averaging amplitude differential equations are analysed numerically.

The book is mainly based on the authors' papers [6–24, 28–39, 115, 125, 156, 2d–27d, 55d, 56d, 62d].

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