

Ram P. Kanwal

ASYMPTOTIC ANALYSIS
A Distributional Approach

1994

Birkhäuser

Boston • Basel • Berlin

Ricardo Estrada
Escuela de Matemática
Universidad de Costa Rica
San José, Costa Rica

Ram P. Kanwal
Department of Mathematics
The Pennsylvania State University
University Park, PA 16802

Library of Congress Cataloging In-Publication Data

Estrada, Ricardo, 1956-

Asymptotic analysis : a distributional approach / Ricardo Estrada
and Ram P. Kanwal.

p. cm.

Includes bibliographical references and index.

ISBN-13: 978-1-4684-0031-1

1. Asymptotic expansions. 2. Theory of distributions (Functional
analysis) I. Kanwal, Ram P., 1924- . II. Title.

QA295.E83 1994

93-36473

515'.234--dc20

CIP

Printed on acid-free paper

© Birkhäuser Boston 1994

Softcover reprint of the hardcover 1st edition 1994

Copyright is not claimed for works of U.S. Government employees.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without prior permission of the copyright owner.

Permission to copy for internal or personal use, or the internal or personal use of specific clients, is granted by Birkhäuser Boston for libraries and other users registered with the Copyright Clearance Center (CCC), provided that the base fee of \$6.00 per copy, plus \$0.20 per page is paid directly to CCC, 21 Congress Street, Salem, MA 01970, U.S.A. Special requests should be addressed directly to Birkhäuser Boston, 675 Massachusetts Avenue, Cambridge, MA 02139, U.S.A.

ISBN-13: 978-1-4684-0031-1 e-ISBN-13: 978-1-4684-0029-8

DOI: 10.1007/978-1-4684-0029-8

Typeset in TEX by the authors.

CONTENTS

CHAPTER 1 Basic Results in Asymptotics

1.1 Introduction.....	1
1.2 Order Symbols	2
1.3 Asymptotic Series	7
1.4 Algebraic and Analytic Operations.....	14
1.5 Existence of Functions with a Given Asymptotic Expansion	19
1.6 Asymptotic Power Series in a Complex Variable	22
1.7 Asymptotic Approximation of Partial Sums.....	28
1.8 The Euler-Maclaurin Summation Formula.....	35

CHAPTER 2 Introduction to the Theory of Distributions

2.1 Introduction.....	43
2.2 The Space of Distributions $\mathcal{D}'$	45
2.3 Algebraic and Analytic Operations.....	49
2.4 Regularization, Pseudofunction and Hadamard Finite Part	53
2.5 Support and Order	60
2.6 Homogeneous Distributions	63
2.7 Distributional Derivatives of Discontinuous Functions	68
2.8 Tempered Distributions and the Fourier Transform.....	74
2.9 Distributions of Rapid Decay	80
2.10 Spaces of Distributions Associated with an Asymptotic Sequence.....	83

CHAPTER 3 A Distributional Theory of Asymptotic Expansions

3.1 Introduction.....	88
3.2 The Taylor Expansion of Distributions.....	89
3.3 The Moment Asymptotic Expansion	94
3.4 Expansions in the Space $\mathcal{P}'$	103
3.5 Laplace's Asymptotic Formula.....	108
3.6 The Method of Steepest Descent.....	113
3.7 Expansion of Oscillatory Kernels	124
3.8 The Expansion of $f(\lambda x)$ as $\lambda \rightarrow \infty$ in Other Cases	131
3.9 Asymptotic Separation of Variables.....	140

CHAPTER 4 The Asymptotic Expansion of Multidimensional Generalized Functions

4.1 Introduction	151
4.2 Taylor Expansion in Several Variables	131
4.3 The Multidimensional Moment Asymptotic Expansion	135
4.4 Laplace's Formula	140
4.5 Fourier Type Integrals	148
4.6 Further Examples	155
4.7 Tensor Products and Partial Asymptotic Expansions	158
4.8 An Application in Quantum Mechanics	163

CHAPTER 5 The Asymptotic Expansion of Certain Series Considered by Ramanujan

5.1 Introduction	195
5.2 Basic Formulas	170
5.3 Lambert Type Series	174
5.4 Distributionally Small Sequences	183
5.5 Multiple Series	191

CHAPTER 6 Series of Dirac Delta Functions

6.1 Introduction	234
6.2 Basic Notions	203
6.3 Several Problems That Lead to Series of Deltas	205
6.4 Dual Taylor Series as Asymptotics of Solutions of Differential Equations	211
6.5 Singular Perturbations	212

REFERENCES	247
-------------------------	------------

INDEX	255
--------------------	------------