Mathematical Aspects
of Classical and Celestial Mechanics

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A.I. Neishtadt

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by A. Iacob

Contents

Preface

This work describes the fundamental principles, problems, and methods of classical mechanics focussing on its mathematical aspects. The authors have striven to give an exposition stressing the working apparatus of classical mechanics, rather than its physical foundations or applications. This apparatus is basically contained in Chapters 1, 3, 4 and 5.

Chapter 1 is devoted to the fundamental mathematical models which are usually employed to describe the motion of real mechanical systems. Special consideration is given to the study of motion under constraints, and also to problems concerned with the realization of constraints in dynamics.

Chapter 3 is concerned with the symmetry groups of mechanical systems and the corresponding conservation laws. Also discussed are various aspects of the theory of the reduction of order for systems with symmetry, often used in applications.

Chapter 4 contains a brief survey of various approaches to the problem of the integrability of the equations of motion, and discusses some of the most general and effective methods of integrating these equations. Various classical examples of integrated problems are outlined. The material presented in this chapter is used in Chapter 5, which is devoted to one of the most fruitful branches of mechanics – perturbation theory. The main task of perturbation theory is the investigation of problems of mechanics which are "close" to exactly integrable problems. Elements of this theory, in particular, the widely used "averaging principle", have emerged in celestial mechanics from attempts to take into account the mutual gravitational perturbations of planets in the solar system.

Chapter 6 is related to Chapters 4 and 5, and studies the theoretical possibility of integrating (in a precisely defined sense) the equations of motion. Approximate integration methods are discussed in Chapter 5: their significance is increased by the fact that integrable systems occur so rarely in reality. Also in this chapter there is a study of the n-body problem with special consideration given to the problem of the stability of the solar system. Some of the classical problems of celestial mechanics are treated in Chapter 2, including the integrable 2-body problem, and the classification of final motions in the 3-body problem. This chapter also contains an analysis of collisions, various aspects of regularization in the general problem of n points interacting gravitationally, and various limiting variants of this problem. Elements of the theory of oscillations are given in Chapter 7.

This text is not a complete exposition of these topics and we do not give detailed proofs. Our main purpose is to acquaint the reader with classical mechanics as a whole, in both its classical and its contemporary aspects. The interested reader will find the necessary proofs, and more detailed information, in the works listed at the end of this volume.