Undergraduate Texts in Mathematics

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M.A. Armstrong

Groups and Symmetry

With 54 Illustrations

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For Jerome and Emily

The beauty of a snow crystal depends on its mathematical regularity and symmetry; but somehow the association of many variants of a single type, all related but no two the same, vastly increases our pleasure and admiration.

D'ARCY THOMPSON
(*On Growth and Form*, Cambridge, 1917.)

En général je crois que les seules structures mathématiques intéressantes, dotées d'une certaine légitimité, sont celles ayant une réalisation naturelle dans le continu.... Du reste, cela se voit très bien dans des théories purement algébriques comme la théorie des groupes abstraits ou on a des groupes plus ou moins étranges apparaissant comme des groupes d'automorphismes de figures continues.

RENÉ THOM
(*Paraboles et Catastrophes*, Flammarion, 1983.)

Preface

Numbers measure size, *groups measure symmetry*. The first statement comes as no surprise; after all, that is what numbers "are for". The second will be exploited here in an attempt to introduce the vocabulary and some of the highlights of elementary group theory.

A word about content and style seems appropriate. In this volume, the emphasis is on *examples* throughout, with a weighting towards the symmetry groups of solids and patterns. Almost all the topics have been chosen so as to show groups in their most natural role, acting on (or permuting) the members of a set, whether it be the diagonals of a cube, the edges of a tree, or even some collection of subgroups of the given group. The material is divided into twenty-eight short chapters, each of which introduces a new result or idea. A glance at the Contents will show that most of the mainstays of a "first course" are here. The theorems of Lagrange, Cauchy, and Sylow all have a chapter to themselves, as do the classification of finitely generated abelian groups, the enumeration of the finite rotation groups and the plane crystallographic groups, and the Nielsen–Schreier theorem.

I have tried to be informal wherever possible, listing only significant results as theorems and avoiding endless lists of definitions. My aim has been to write a book which can be read with or without the support of a course of lectures. It is not designed for use as a dictionary or handbook, though new concepts are shown in bold type and are easily found in the index. Every chapter ends with a collection of exercises designed to consolidate, and in some cases fill out, the main text. It is essential to work through as many of these as possible before moving from one chapter to the next. Mathematics is not for spectators; to gain in understanding, confidence, and enthusiasm one has to participate.

As prerequisites I assume a first course in linear algebra (including matrix multiplication and the representation of linear maps between Euclidean spaces by matrices, though not the abstract theory of vector spaces) plus familiarity with the basic properties of the real and complex numbers. It would seem a pity to teach group theory without matrix groups available as a rich source of examples, especially since matrices are so heavily used in applications.

Elementary material of this type is all common stock, nevertheless it is not static, and improvements are made from time to time. Three such should be mentioned here: H. Wielandt's approach to the Sylow theorems (Chapter 20), J. McKay's proof of Cauchy's theorem (Chapter 13), and the introduction of groups acting on trees by J.-P. Serre (Chapter 28). Another influence is of a more personal nature. As a student I had the good fortune to study with A.M. Macbeath, whose lectures first introduced me to group theory. The debt of gratitude from pupil to teacher is best paid in kind. If this little book can pass on something of the same appreciation of the beauty of mathematics as was shown to me, then I shall be more than satisfied.

Durham, England M.A.A.
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