Undergraduate Texts in Mathematics

Springer Science+Business Media, LLC

Undergraduate Texts in Mathematics

Abbott: Understanding Analysis.
Anglin: Mathematics: A Concise History and Philosophy.
Readings in Mathematics.
Anglin/Lambek: The Heritage of Thales.
Readings in Mathematics.
Apostol: Introduction to Analytic Number Theory. Second edition.
Armstrong: Basic Topology.
Armstrong: Groups and Symmetry.
Axler: Linear Algebra Done Right. Second edition.
Beardon: Limits: A New Approach to Real Analysis.
Bak/Newman: Complex Analysis. Second edition.
Banchoff/Wermer: Linear Algebra Through Geometry. Second edition.
Berberian: A First Course in Real Analysis.
Bix: Conics and Cubics: A Concrete Introduction to Algebraic Curves.
Brémaud: An Introduction to Probabilistic Modeling.
Bressoud: Factorization and Primality Testing.
Bressoud: Second Year Calculus.
Readings in Mathematics.
Brickman: Mathematical Introduction to Linear Programming and Game Theory.
Browder: Mathematical Analysis: An Introduction.
Buchmann: Introduction to Cryptography.
Buskes/van Rooij: Topological Spaces: From Distance to Neighborhood.
Callahan: The Geometry of Spacetime: An Introduction to Special and General Relavitity.
Carter/van Brunt: The Lebesgue–Stieltjes Integral: A Practical Introduction.
Cederberg: A Course in Modern Geometries. Second edition.
Childs: A Concrete Introduction to Higher Algebra. Second edition.
Chung: Elementary Probability Theory with Stochastic Processes. Third edition.
Cox/Little/O'Shea: Ideals, Varieties, and Algorithms. Second edition.
Croom: Basic Concepts of Algebraic Topology.
Curtis: Linear Algebra: An Introductory Approach. Fourth edition.
Devlin: The Joy of Sets: Fundamentals of Contemporary Set Theory. Second edition.
Dixmier: General Topology.
Driver: Why Math?
Ebbinghaus/Flum/Thomas: Mathematical Logic. Second edition.
Edgar: Measure, Topology, and Fractal Geometry.
Elaydi: An Introduction to Difference Equations. Second edition.
Exner: An Accompaniment to Higher Mathematics.
Exner: Inside Calculus.
Fine/Rosenberger: The Fundamental Theory of Algebra.
Fischer: Intermediate Real Analysis.
Flanigan/Kazdan: Calculus Two: Linear and Nonlinear Functions. Second edition.
Fleming: Functions of Several Variables. Second edition.
Foulds: Combinatorial Optimization for Undergraduates.
Foulds: Optimization Techniques: An Introduction.
Franklin: Methods of Mathematical Economics.
Frazier: An Introduction to Wavelets Through Linear Algebra.
Gamelin: Complex Analysis.
Gordon: Discrete Probability.
Hairer/Wanner: Analysis by Its History.
Readings in Mathematics.

(continued after index)

M. A. Armstrong

Basic Topology

With 132 Illustrations

Springer

M. A. Armstrong
Department of Mathematics
University of Durham
Durham DH1 3LE
England

Mathematics Subject Classification (2000): 55-01, 54-01

Library of Congress Cataloging-in-Publication Data
Armstrong, M. A. (Mark Anthony)
Basic topology.
(Undergraduate texts in mathematics)
Bibliography: p.
Includes index.
1. Topology. I. Title. II. Series.
QA611.A68 1983 514 83-655

Original edition published in 1979 by McGraw-Hill Book Company (UK) Ltd., Maidenhead, Berkshire, England.

ISBN 978-1-4419-2819-1 ISBN 978-1-4757-1793-8 (eBook) Printed on acid-free paper.
DOI 10.1007/978-1-4757-1793-8

9 8 SPIN 11007081

springeronline.com

Dedicated to the memory of

PAUL DUFÉTELLE

Preface to the Springer Edition

This printing is unchanged, though the opportunity has been taken to correct one or two misdemeanours. In particular Problems 2.13, 3.13 and 3.19 are now correctly stated, and Tietze has regained his final "e". My thanks go to Professor P. R. Halmos and to Springer-Verlag for the privilege of appearing in this series.

M.A.A.
Durham, January 1983.

Preface

This is a topology book for undergraduates, and in writing it I have had two aims in mind. Firstly, to make sure the student sees a variety of different techniques and applications involving point set, geometric, and algebraic topology, without delving too deeply into any particular area. Secondly, to develop the reader's geometrical insight; topology is after all a branch of geometry.

The prerequisites for reading the book are few, a sound first course in real analysis (as usual!), together with a knowledge of elementary group theory and linear algebra. A reasonable degree of 'mathematical maturity' is much more important than any previous knowledge of topology.

The layout is as follows. There are ten chapters, the first of which is a short essay intended as motivation. Each of the other chapters is devoted to a single important topic, so that identification spaces, the fundamental group, the idea of a triangulation, surfaces, simplicial homology, knots and covering spaces, all have a chapter to themselves.

Some motivation is surely necessary. A topology book at this level which begins with a set of axioms for a topological space, as if these were an integral part of nature, is in my opinion doomed to failure. On the other hand, topology should not be presented as a collection of party tricks (colouring knots and maps, joining houses to public utilities, or watching a fly escape from a Klein bottle). These things all have their place, but they must be shown to fit into a unified mathematical theory, and not remain dead ends in themselves. For this reason, knots appear at the end of the book, and not at the beginning. It is not the knots which are so interesting, but rather the variety of techniques needed to deal with them.

Chapter 1 begins with Euler's theorem for polyhedra, and the theme of the book is the search for topological invariants of spaces, together with techniques for calculating them. Topology is complicated by the fact that something which is, by its very nature, topologically invariant is usually hard to calculate, and vice versa the invariance of a simple number like the Euler characteristic can involve a great deal of work.

The balance of material was influenced by the maxim that a theory and its payoff in terms of applications should, wherever possible, be given equal weight. For example, since homology theory is a good deal of trouble to set up (a whole chapter), it must be shown to be worth the effort (a whole chapter of applications). Moving away from a topic is always difficult, and the temptation to include more and more is hard to resist. But to produce a book of reasonable length some topics just have to go; I mention particularly in this respect the omission of any systematic method for calculating homology groups. In formulating definitions, and choosing proofs, I have not always taken the shortest path. Very often the version of a definition or result which is most convenient to work with, is not at all natural at first sight, and this is above all else a book for beginners.

Most of the material can be covered in a one-year course at third-year (English) undergraduate level. But there is plenty of scope for shorter courses involving a selection of topics, and much of the first half of the book can be taught to second-year students. Problems are included at the end of just about every section, and a short bibliography is provided with suggestions for parallel reading and as to where to go next.

The material presented here is all basic and has for the most part appeared elsewhere. If I have made any contribution it is one of selection and presentation.

Two topics deserve special mention. I first learned about the Alexander polynomial from J. F. P. Hudson, and it was E. C. Zeeman who showed me how to do surgery on surfaces. To both of them, and particularly to Christopher Zeeman for his patience in teaching me topology, I offer my best thanks.

I would also like to thank R. S. Roberts and L. M. Woodward for many useful conversations, Mrs J. Gibson for her speed and skill in producing the manuscript, and Cambridge University Press for permission to reproduce the quotation from Hardy's 'A Mathematician's Apology' which appears at the beginning of Chapter 1. Finally, a special word of thanks to my wife Anne Marie for her constant encouragement.

M.A.A.
Durham, July 1978.

Contents