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Approximation of Functions
of Several Variables
and Imbedding Theorems

Translated from the Russian by
J. M. Danskin



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