

# Contents

Preface — v

## Introduction — 1

### 0 Prerequisites — 7

- 0.1 The Lebesgue integral — 7
- 0.2 Some functional analysis — 18
- 0.3 Basic function spaces — 25
- 0.4 Comments on Chapter 0 — 43
- 0.5 Exercises to Chapter 0 — 46

### 1 Classical BV-spaces — 55

- 1.1 Functions of bounded variation — 55
- 1.2 Bounded variation and continuity — 71
- 1.3 Functions of bounded Wiener variation — 84
- 1.4 Functions of several variables — 91
- 1.5 Comments on Chapter 1 — 100
- 1.6 Exercises to Chapter 1 — 104

### 2 Nonclassical BV-spaces — 112

- 2.1 The Wiener–Young variation — 112
- 2.2 The Waterman variation — 125
- 2.3 The Schramm variation — 152
- 2.4 The Riesz–Medvedev variation — 161
- 2.5 The Korenblum variation — 169
- 2.6 Higher order Wiener-type variations — 182
- 2.7 Comments on Chapter 2 — 187
- 2.8 Exercises to Chapter 2 — 202

### 3 Absolutely continuous functions — 208

- 3.1 Continuity and absolute continuity — 208
- 3.2 The Vitali–Banach–Zaretskij theorem — 211
- 3.3 Reconstructing a function from its derivative — 218
- 3.4 Rectifiable functions — 231
- 3.5 The Riesz–Medvedev theorem — 240
- 3.6 Higher order Riesz-type variations — 244
- 3.7 Comments on Chapter 3 — 249
- 3.8 Exercises to Chapter 3 — 260

**4 Riemann–Stieltjes integrals — 268**

- 4.1 Classical RS-integrals — 268
- 4.2 Bounded variation and duality — 292
- 4.3 Bounded  $p$ -variation and duality — 298
- 4.4 Nonclassical RS-integrals — 302
- 4.5 Comments on Chapter 4 — 311
- 4.6 Exercises to Chapter 4 — 316

**5 Nonlinear composition operators — 324**

- 5.1 The composition operator problem — 324
- 5.2 Boundedness and continuity — 344
- 5.3 Spaces of differentiable functions — 354
- 5.4 Global Lipschitz continuity — 364
- 5.5 Local Lipschitz continuity — 368
- 5.6 Comments on Chapter 5 — 377
- 5.7 Exercises to Chapter 5 — 382

**6 Nonlinear superposition operators — 385**

- 6.1 Boundedness and continuity — 385
- 6.2 Lipschitz continuity — 400
- 6.3 Uniform boundedness and continuity — 406
- 6.4 Functions of several variables — 415
- 6.5 Comments on Chapter 6 — 419
- 6.6 Exercises to Chapter 6 — 423

**7 Some applications — 425**

- 7.1 Convergence criteria for Fourier series — 425
- 7.2 Fourier series and Waterman spaces — 429
- 7.3 Applications to nonlinear integral equations — 435
- 7.4 Comments on Chapter 7 — 444

References — 453

List of functions — 467

List of symbols — 468

Index — 472