Contents

Introduction 1

Chapter 1. Classical Volterra equations of the first kind 5
1.1. Classification of integral Volterra equations of the fist kind 5
1.2. The Gronwall–Bellman lemma 10
1.3. A difference analog of the Gronwall–Bellman lemma 14
1.4. Self-regularization 17
1.5. Two-parametric (α, h)-regularization 19
1.6. Inequalities with isotone operators 32
1.7. Inequalities with interchangeable isotone operators 39
1.8. Unimprovable estimates of solutions of multidimensional integral inequalities 43
1.9. The well-posedness of a two-dimensional Volterra equation of the first kind 49
1.10. Unimprovable estimates of solutions of two-dimensional difference inequalities 51

Chapter 2. Volterra equations of the first kind with two variable integration limits. The case $a(t_0) < t_0$ 57
2.1. Problem statement 57
2.2. The method of steps 58
2.3. Illustrative examples 62
2.4. The existence and uniqueness theorem 65
2.5. An estimate of the solution stability 68
2.6. The study of a special problem of mathematical programming . . 70

2.7. A numerical solution of the test example 79
2.8. A geometrical illustration of the reduction by unity in the order of convergence 81
2.9. A theorem on the convergence of the quadrature method (the general case) 84
2.10. Some numerical results 90
2.11. On self-regularization 96

Chapter 3. Volterra equations of the first kind with two variable limits of integration. The case $a(t_0) = t_0$ **99**
3.1. Problem statement 99
3.2. Solution of the simplest test equation 100
3.3. Existence and uniqueness theorem (the general case) 102
3.4. Estimation of the solution stability 104
3.5. Some generalizations of the Gronwall–Bellman inequality 107
3.6. Numerical solution of a test example 114
3.7. The proof of convergence for the quadrature method (the general case) 118
3.8. Some numerical results 124
3.9. Self-regularization (the case of a disturbance in the right-hand side) 126
3.10. Stability of a numerical solution with respect to disturbances of $a(t)$ 128
3.11. Multidimensional Volterra equations of the first kind related to the modelling of nonlinear dynamic systems using the Volterra series 130

Bibliography **155**

Index **167**