Vladimir V. Andrievskii Hans-Peter Blatt

Discrepancy of Signed Measures and Polynomial Approximation

Springer

Vladimir V. Andrievskii
Department of Mathematics and Computer Science
Kent State University
Kent, OH 44242
USA
andriyev@mcs.kent.edu

Hans-Peter Blatt
Mathematisch-Geographische Fakultät
Katholische Universität Eichstätt
D-85072 Eichstätt
Germany
hans.blatt@ku-eichstaett.de

Mathematics Subject Classification (2000): 26C10, 31A15, 30E10, 42CO5, 4

Library of Congress Cataloging-in-Publication Data
Andrievskii, V. V., 1953–
Discrepancy of signed measures and polynomial approximation / Vladimir V. Andrievskii, Hans-Peter Blatt.
p. cm.— (Springer monographs in mathematics)
Includes bibliographical references and index.
ISBN 978-1-4419-3146-7 ISBN 978-1-4757-4999-1(eBook)
DOI 10.1007/978-1-4757-4999-1
1. Approximation theory. 2. Orthogonal polynomials. I. Blatt, Hans-Peter. II. Title. III. Series.
QA221 .A3567 2001
511.4—dc21 2001032837

Printed on acid-free paper.

© 2002 Springer Science+Business Media New York
Originally published by Springer-Verlag New York, Inc. in 2002
Softcover reprint of the hardcover 1st edition 2002
All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher Springer Science+Business Media, LLC.
except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.
The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Steven Pisano; manufacturing supervised by Erica Bresler.
Photocomposed pages prepared from the authors' LaTeX files.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4419-3146-7 SPIN 10696722

Contents

Preface **vii**

1 Auxiliary Facts **1**
1.1 Basic Potential-Theoretical Concepts 1
1.1.1 Sets and Curves 1
1.1.2 Harmonic and Subharmonic Functions, Maximum Principle 2
1.1.3 Green's Formula 6
1.1.4 Harmonic Measure 8
1.1.5 Energy, Capacity, Equilibrium Measure 10
1.1.6 Maximum Principle 14
1.1.7 Weak* Topology, Helly's Theorem 14
1.1.8 Green's Function 14
1.1.9 Fekete Points, Transfinite Diameter, Chebyshev Constant 17
1.2 Conformal and Quasiconformal Mappings 19
1.2.1 Basic Principles of Complex Analysis 19
1.2.2 Equilibrium Measure and Conformal Mappings . . . 21
1.2.3 Distortion Theorems 22
1.2.4 Module of a Family of Curves and Arcs 24
1.2.5 Quasiconformal Curves and Arcs 27
1.2.6 Dini-Smooth Curves and Arcs 32
1.3 Faber Polynomials and Grunsky Coefficients 37
1.4 Monic Polynomials 40
1.5 Linear Approximation 44
1.5.1 Polynomial Approximation 45
1.5.2 Averaging and Smoothing 46
1.6 Ordering Symbols 48
1.7 Historical Comments 48

2 Zero Distribution of Polynomials **49**
2.1 Jentzsch–Szegő Type Theorems 50
2.2 Erdős–Turán Type Theorems for Quasiconformal Curves and Arcs 56
2.2.1 Polynomial Bounds on Quasiconformal Curves . . . 57
2.2.2 Polynomial Bounds on Quasiconformal Arcs 68
2.3 Sharpness and Extensions of Erdős–Turán Type Theorems . 72
2.4 Erdős–Turán Type Theorems on Compact Sets with Smooth Curves and Arcs 78
2.5 Erdős–Turán Type Theorems on a System of Intervals . . . 86
2.6 Historical Comments 92

3 Discrepancy Theorems via Two–Sided Bounds for Potentials **95**
3.1 Estimates for Quasiconformal Curves and Arcs 96
3.2 Local Estimates for Intervals 108
3.3 Local Estimates for Dini–Smooth Arcs and Curves 114
3.4 Historical Comments 128

4 Discrepancy Theorems via One-Sided Bounds for Potentials **129**
4.1 Outer Bounds for Potentials 130
4.2 Inner and Outer Bounds for Potentials 150
4.3 Inner Bounds for Potentials of Signed Measures on Analytic Jordan Curves 152
4.4 Another Approach for Dini-Smooth Arcs 161
4.5 Historical Comments 172

5 Discrepancy Theorems via Energy Integrals **173**
5.1 Modulus of a Doubly Split Plane 174
5.2 Estimates of Mass Distributions from Their Energies 178
5.3 Historical Comments 186

6 Applications of Jentzsch–Szegő and Erdős–Turán Type Theorems **187**
6.1 Polynomials of Best Uniform Approximation 188
6.2 Polynomials of Near-Best Approximation 193
6.3 Polynomials of Maximal Convergence 202
6.4 a-Values of Orthogonal Polynomials on Quasidisks 208
6.5 Zeros of Bieberbach Polynomials and Their Derivatives . . . 215
6.6 Historical Comments 223

7 Applications of Discrepancy Theorems **225**
7.1 Distribution of Fekete Points 226
7.2 Fekete Points for Domains with Analytic Boundary 229

7.3 Extreme Points in Uniform Polynomial Approximation . . . 241
7.4 Alternation Points in Chebyshev Approximation 248
7.5 Sign Changes in L^p-Approximation 254
7.5.1 The Case $1 < p < \infty$ 256
7.5.2 The Case $p = 1$ 266
7.6 Extremal Polynomials on the Real Line 274
7.7 Zeros of Orthogonal Polynomials 297
7.7.1 Orthogonal Polynomials on Real Intervals 297
7.7.2 Orthogonal Polynomials on the Unit Circle 312
7.8 Historical Comments 317

8 Special Topics **321**
8.1 Polynomial Approximation to a Piecewise Analytic Function on Touching Domains 321
8.2 Uniform Convergence of Bieberbach Polynomials in Domains with Quasiconformal Boundary 336
8.3 Historical Comments 340

A Conformally Invariant Characteristics of Curve Families **341**
A.1 Module and Extremal Length of a Curve Family 341
A.2 Reduced Module 357

B Basics in the Theory of Quasiconformal Mappings **361**
B.1 Quasiconformal Mappings 361
B.2 Quasiconformal Curves and Arcs 369

C Constructive Theory of Functions of a Complex Variable **381**
C.1 Jackson Type Kernels 383
C.2 Polynomial Kernels Approximating the Cauchy Kernel . . . 386
C.3 Inverse Theorems 393
C.4 Polynomial Approximation in Domains with Smooth Boundary 403

D Miscellaneous Topics **407**
D.1 The Regularized Distance 407
D.2 Green's Function for a System of Intervals 409

Bibliography **415**

Notation **427**

Index **435**