

Graduate Texts in Mathematics **13**

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Graduate Texts in Mathematics

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continued after index

Frank W. Anderson Kent R. Fuller

Rings and Categories of Modules

Second Edition



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Frank W. Anderson
University of Oregon
Department of Mathematics
Eugene, OR 97403
USA

Kent R. Fuller
University of Iowa
Department of Mathematics
Iowa City, IA 52242
USA

Editorial Board

J.H. Ewing
Department of
Mathematics
Indiana University
Bloomington, IN 47405
USA

F.W. Gehring
Department of
Mathematics
University of Michigan
Ann Arbor, MI 48109
USA

P.R. Halmos
Department of
Mathematics
Santa Clara University
Santa Clara, CA 95053
USA

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Preface

This book is intended to provide a reasonably self-contained account of a major portion of the general theory of rings and modules suitable as a text for introductory and more advanced graduate courses. We assume the familiarity with rings usually acquired in standard undergraduate algebra courses. Our general approach is categorical rather than arithmetical. The continuing theme of the text is the study of the relationship between the one-sided ideal structure that a ring may possess and the behavior of its categories of modules.

Following a brief outline of set-theoretic and categorical foundations, the text begins with the basic definitions and properties of rings, modules and homomorphisms and ranges through comprehensive treatments of direct sums, finiteness conditions, the Wedderburn-Artin Theorem, the Jacobson radical, the hom and tensor functions, Morita equivalence and duality, decomposition theory of injective and projective modules, and semiperfect and perfect rings. In this second edition we have included a chapter containing many of the classical results on artinian rings that have helped to form the foundation for much of the contemporary research on the representation theory of artinian rings and finite dimensional algebras. Both to illustrate the text and to extend it we have included a substantial number of exercises covering a wide spectrum of difficulty. There are, of course, many important areas of ring and module theory that the text does not touch upon. For example, we have made no attempt to cover such subjects as homology, rings of quotients, or commutative ring theory.

This book has evolved from our lectures and research over the past several years. We are deeply indebted to many of our students and colleagues for their ideas and encouragement during its preparation. We extend our sincere thanks to them and to the several people who have helped with the preparation of the manuscripts for the first two editions, and/or pointed out errors in the first.

Finally, we apologize to the many authors whose works we have used but not specifically cited. Virtually all of the results in this book have appeared in some form elsewhere in the literature, and they can be found either in the books and articles that are listed in our bibliography, or in those listed in the collective bibliographies of our citations.

Eugene, OR
Iowa City, IA

Frank W. Anderson
Kent R. Fuller
January 1992

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