

Kirsti Andersen

# The Geometry of an Art

The History of the Mathematical  
Theory of Perspective from Alberti  
to Monge

 Springer

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