

Tuomas Hytönen • Jan van Neerven
Mark Veraar • Lutz Weis

Analysis in Banach Spaces

Volume III: Harmonic Analysis
and Spectral Theory

 Springer

Contents

11	Singular integral operators	1
11.1	Local oscillations of functions	2
11.1.a	Sparse collections and Lerner's formula	4
11.1.b	Almost orthogonality in L^p	9
11.1.c	Maximal oscillatory norms for L^p spaces	13
11.1.d	The dyadic Hardy space and BMO	17
11.2	Singular integrals and extrapolation of L^{p_0} bounds	22
11.2.a	Calderón–Zygmund decomposition and case $p \in (1, p_0)$	25
11.2.b	Local oscillations of Tf and case $p \in (p_0, \infty)$	29
11.2.c	The action of singular integrals on L^∞	34
11.3	Calderón–Zygmund operators and sparse bounds	41
11.3.a	An abstract domination theorem	45
11.3.b	Sparse operators and domination	50
11.3.c	Sparse domination of Calderón–Zygmund operators	54
11.3.d	Weighted norm inequalities and the A_2 theorem	56
11.3.e	Sharpness of the A_2 theorem	66
11.4	Notes	70
12	Dyadic operators and the $T(1)$ theorem	87
12.1	Dyadic singular integral operators	88
12.1.a	Haar multipliers	89
12.1.b	Nested collections of unions of dyadic cubes	102
12.1.c	The elementary operators of Figiel	108
12.2	Paraproducts	122
12.2.a	Necessary conditions for boundedness	125
12.2.b	Sufficient conditions for boundedness	134
12.2.c	Symmetric paraproducts	140
12.2.d	Mei's counterexample: no simple sufficient conditions	145
12.3	The $T(1)$ theorem for abstract bilinear forms	147
12.3.a	Weakly defined bilinear forms	148
12.3.b	The BCR algorithm and Figiel's decomposition	154

12.3.c	Figiel's $T(1)$ theorem	160
12.3.d	Improved estimates via random dyadic cubes	167
12.4	The $T(1)$ theorem for singular integrals	174
12.4.a	Consequences of the $T(1)$ theorem	185
12.4.b	The dyadic representation theorem	196
12.5	Notes	208
13	The Fourier transform and multipliers	225
13.1	Bourgain's theorem on Fourier type	226
13.1.a	Hinrichs's inequality: breaking the trivial bound	231
13.1.b	The finite Fourier transform and sub-multiplicativity	237
13.1.c	Key lemmas for an initial uniform bound	244
13.1.d	Conclusion via duality and interpolation	251
13.2	Fourier multipliers as singular integrals	258
13.2.a	Smooth multipliers have Calderón–Zygmund kernels	260
13.2.b	Mihlin multipliers have Hörmander kernels	267
13.3	Necessity of UMD for multiplier theorems	273
13.4	Notes	283
14	Function spaces	293
14.1	Summary of the main results	295
14.2	Preliminaries	300
14.2.a	Notation	300
14.2.b	A density lemma and Young's inequality	303
14.2.c	Inhomogeneous Littlewood–Paley sequences	305
14.3	Interpolation of L^p -spaces with change of weights	310
14.3.a	Complex interpolation	311
14.3.b	Real interpolation	314
14.4	Besov spaces	319
14.4.a	Definitions and basic properties	320
14.4.b	Fourier multipliers	326
14.4.c	Embedding theorems	332
14.4.d	Difference norms	337
14.4.e	Interpolation	344
14.4.f	Duality	348
14.5	Besov spaces, random sums, and multipliers	352
14.5.a	The Fourier transform on Besov spaces	356
14.5.b	Smooth functions have R -bounded ranges	360
14.6	Triebel–Lizorkin spaces	365
14.6.a	The Peetre maximal function	365
14.6.b	Definitions and basic properties	370
14.6.c	Fourier multipliers	373
14.6.d	Embedding theorems	375
14.6.e	Difference norms	380
14.6.f	Interpolation	384

14.6.g	Duality	387
14.6.h	Pointwise multiplication by $\mathbf{1}_{\mathbb{R}_+}$ in $B_{p,q}^s$ and $F_{p,q}^s$	387
14.7	Bessel potential spaces	393
14.7.a	General embedding theorems	393
14.7.b	Embedding theorems under geometric conditions	395
14.7.c	Interpolation	401
14.7.d	Pointwise multiplication by $\mathbf{1}_{\mathbb{R}_+}$ in $H^{s,p}$	404
14.8	Notes	407
15	Extended calculi and powers of operators	419
15.1	Extended calculi	419
15.1.a	The primary calculus	420
15.1.b	The extended Dunford calculus	424
15.1.c	Extended calculus via compensation	435
15.2	Fractional powers	436
15.2.a	Definition and basic properties	437
15.2.b	Representation formulas	445
15.3	Bounded imaginary powers	456
15.3.a	Definition and basic properties	457
15.3.b	Identification of fractional domain spaces	459
15.3.c	Connections with sectoriality	462
15.3.d	Connections with almost γ -sectoriality	470
15.3.e	Connections with γ -sectoriality	472
15.3.f	Connections with boundedness of the H^∞ -calculus	476
15.3.g	The Hilbert space case	482
15.3.h	Examples	483
15.4	Strip type operators	484
15.4.a	Nollau's theorem	485
15.4.b	Monniaux's theorem	489
15.4.c	The Dore–Venni theorem	494
15.5	The bisectorial H^∞ -calculus revisited	497
15.5.a	Spectral projections	498
15.5.b	Sectoriality versus bisectoriality	500
15.6	Notes	502
16	Perturbations and sums of operators	515
16.1	Sums of unbounded operators	515
16.2	Perturbation theorems	516
16.2.a	Perturbations of sectorial operators	517
16.2.b	Perturbations of the H^∞ -calculus	523
16.3	Sum-of-operator theorems	533
16.3.a	The sum of two sectorial operators	536
16.3.b	Operator-valued H^∞ -calculus and closed sums	541
16.3.c	The joint H^∞ -calculus	546
16.3.d	The absolute calculus and closed sums	551

16.3.e	The absolute calculus and real interpolation	553
16.4	Notes	560
17	Maximal regularity	569
17.1	The abstract Cauchy problem	570
17.2	Maximal L^p -regularity	575
17.2.a	Definition and basic properties	575
17.2.b	The initial value problem	583
17.2.c	The role of semigroups	586
17.2.d	Uniformly exponentially stable semigroups	595
17.2.e	Permanence properties	598
17.2.f	Maximal continuous regularity	613
17.2.g	Perturbation and time-dependent problems	619
17.3	Characterisations of maximal L^p -regularity	626
17.3.a	Fourier multiplier approach	626
17.3.b	The end-point cases $p = 1$ and $p = \infty$	633
17.3.c	Sum-of-operators approach	636
17.3.d	Maximal L^p -regularity on the real line	644
17.4	Examples and counterexamples	657
17.4.a	The heat semigroup and the Poisson semigroup	657
17.4.b	End-point maximal regularity versus containment of c_0	660
17.4.c	Analytic semigroups may fail maximal regularity	665
17.5	Notes	671
18	Nonlinear parabolic evolution equations in critical spaces	689
18.1	Semi-linear evolution equations with $F = F_{\text{Tr}}$	692
18.2	Local well-posedness for quasi-linear evolution equations	697
18.2.a	Setting	697
18.2.b	Main local well-posedness result	701
18.2.c	Proof of the main result	702
18.2.d	Maximal solutions	712
18.3	Examples and comparison	717
18.4	Long-time existence for small initial data and $F = F_c$	725
18.5	Notes	727
Q	Questions	733
K	Measurable semigroups	745
K.1	Measurable semigroups	745
K.2	Uniform exponential stability	754
K.3	Analytic semigroups	756
K.4	An interpolation result	758
K.5	Notes	762

L	The trace method for real interpolation	763
L.1	Preliminaries	763
L.2	The trace method	765
L.3	Reiteration	771
L.4	Mixed derivatives and Sobolev embedding	774
	L.4.a Results for the half-line	775
	L.4.b Extension operators	779
	L.4.c Results for bounded intervals	780
L.5	Notes	782
	References	783
	Index	820