

An Introduction To Probability Theory And Its Applications. Volume 2

William Feller – 2. ed. - ©1971

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