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978-0-521-86320-9 - Nonlinear Analysis and Semilinear Elliptic Problems

Antonio Ambrosetti and Andrea Malchiodi

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